ME3760 Dynamics II, Spring 1998
Formula Sheet, Final Exam
Dr. Ferri

**Chapter 6**
**General:**

$${}^{B}\dot{\vec{Q}} = {}^{A}\dot{\vec{Q}} + \vec{\omega}_{B/A} \times \vec{Q} \quad ; \quad \vec{\omega}_{A/C} = \vec{\omega}_{A/B} + \vec{\omega}_{B/C} \quad ; \quad \vec{\alpha}_{A/G} = {}^{G}\dot{\vec{\omega}}_{A/G} = {}^{B}\dot{\vec{\omega}}_{A/G} + \vec{\omega}_{B/G} \times \vec{\omega}_{A/G}$$

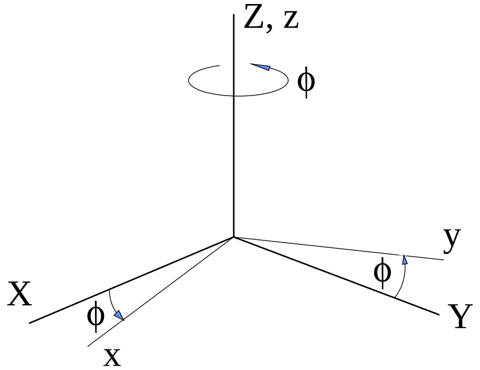
**Point P moving with respect to body B:**

$$\vec{v}_{P/F} = \vec{v}_{P/B} + \vec{v}_{O'/F} + \vec{\omega}_{B/F} \times \vec{r}_{O'P}$$
$$\vec{a}_{P/F} = \vec{a}_{P/B} + \vec{a}_{O'/F} + \vec{\alpha}_{B/F} \times \vec{r}_{O'P} + \vec{\omega}_{B/F} \times \left(\vec{\omega}_{B/F} \times \vec{r}_{O'P}\right) + 2\,\vec{\omega}_{B/F} \times \vec{v}_{P/B}$$

**Points P and O′ both on rigid body B:**

$$\vec{v}_{P/F} = \vec{v}_{O'/F} + \vec{\omega}_{B/F} \times \vec{r}_{O'P} \quad ; \quad \vec{a}_{P/F} = \vec{a}_{O'/F} + \vec{\alpha}_{B/F} \times \vec{r}_{O'P} + \vec{\omega}_{B/F} \times \left(\vec{\omega}_{B/F} \times \vec{r}_{O'P}\right)$$

**Rotational Transformation Matrices:**




$$\{Q\}_{\mathscr{B}} = [T_z(\phi)]\,\{Q\}_{\mathscr{F}}$$

$$\left[T_z(\phi)\right] = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**Chapter 7:**

$$I^P_{xx} = \int_B \left(y^2 + z^2\right) dm; \quad I^P_{yy} = \int_B \left(x^2 + z^2\right) dm; \quad I^P_{zz} = \int_B \left(x^2 + y^2\right) dm$$

$$I^P_{xy} = -\int_B xy\ dm; \quad I^P_{xz} = -\int_B xz\ dm; \quad I^P_{yz} = -\int_B yz\ dm$$

$$I^P_{xx} = I^C_{xx} + m\left(\bar{y}^2 + \bar{z}^2\right) \qquad I^P_{xy} = I^C_{xy} - m\,\bar{x}\,\bar{y} \qquad [I'] = [T][I][T]^T$$

where $[I']$ and [I] are the inertia matrices relative to coordinates $x'y'z'$ and xyz, respectively, and [T] is a rotational transformation matrix that relates the components of any vector in the two frames:

$$\begin{Bmatrix} Q_{x'} \\ Q_{y'} \\ Q_{z'} \end{Bmatrix} = [T] \begin{Bmatrix} Q_x \\ Q_y \\ Q_z \end{Bmatrix}$$

## Angular Momentum:

$$\{H_C\} = [I^C]\{\omega_{B/G}\}$$

$$\vec{H}_C = \left(I^C_{xx}\,\omega_x + I^C_{xy}\,\omega_y + I^C_{xz}\,\omega_z\right)\hat{i} + \left(I^C_{xy}\,\omega_x + I^C_{yy}\,\omega_y + I^C_{yz}\,\omega_z\right)\hat{j} + \left(I^C_{xz}\,\omega_x + I^C_{yz}\,\omega_y + I^C_{zz}\,\omega_z\right)\hat{k}$$

For principal axes, the angular momentum can be written:

$$\vec{H}_C = I^C_{xx}\,\omega_x\ \hat{i} + I^C_{yy}\,\omega_y\ \hat{j} + I^C_{zz}\,\omega_z\ \hat{k}$$

## Newton and Euler's Laws and Equations for Rigid Bodies:

In general, the sum of the moments about a body's center of mass is given by:

$$\sum \vec{M}_C = {}^G\dot{\vec{H}}_C = {}^B\dot{\vec{H}}_C + \vec{\omega}_{B/G} \times \vec{H}_C$$

Euler's Equations for body-fixed, principal axes:

$$\Sigma M_{C_x} = I^C_{xx}\ \alpha_x - \left(I^C_{yy} - I^C_{zz}\right)\omega_y\,\omega_z$$
$$\Sigma M_{C_y} = I^C_{yy}\ \alpha_y - \left(I^C_{zz} - I^C_{xx}\right)\omega_z\,\omega_x$$
$$\Sigma M_{C_z} = I^C_{zz}\ \alpha_z - \left(I^C_{xx} - I^C_{yy}\right)\omega_x\,\omega_y$$

Newton's Second Law for a rigid body or collection of particles:

$$\Sigma\vec{F} = m\,\vec{a}_C$$

## Impulse-Momentum Relations:

$$\int_{t_1}^{t_2} \Sigma\vec{F}\,dt = (m\vec{v}_C)_2 - (m\vec{v}_C)_1 = \vec{L}_2 - \vec{L}_1$$

$$\int_{t_1}^{t_2} \Sigma\vec{M}_C\,dt = \left(\vec{H}_C\right)_2 - \left(\vec{H}_C\right)_1 \quad ; \quad \int_{t_1}^{t_2} \Sigma\vec{M}_O\,dt = \left(\vec{H}_O\right)_2 - \left(\vec{H}_O\right)_1 \quad \text{for a fixed point O}$$

Conservation of Linear and angular momentum: Suppose that there are no forces in the x-direction; then $(L_x)_2 = (L_x)_1$. Likewise, suppose that there are no moments about points C or O in the z-direction; then $\left(H_{C_z}\right)_2 = \left(H_{C_z}\right)_1$ or $\left(H_{O_z}\right)_2 = \left(H_{O_z}\right)_1$, respectively.

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**Work-Energy Relations:**

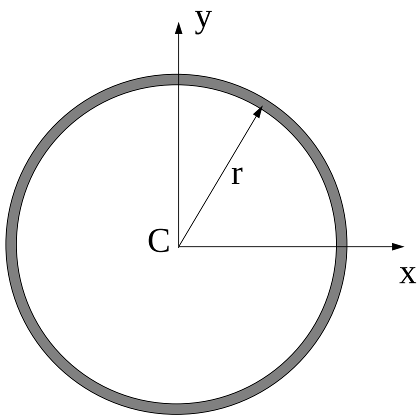
$$T = \frac{1}{2} m \vec{v}_C \cdot \vec{v}_C + \frac{1}{2} \vec{\omega} \cdot \vec{H}_C \quad ; \qquad T = \frac{1}{2} \vec{\omega} \cdot \vec{H}_O \quad \text{for a fixed point O}$$

$$W_{1\rightarrow 2} = T_2 - T_1 \quad \textbf{or} \quad W_{1\rightarrow 2}^{NC} = (T_2 + V_2) - (T_1 + V_1)$$

where the potential energy V accounts for all conservative forces (such as spring and gravity forces) and $W^{NC}$ is the work done by any nonconservative forces (such as externally applied forces/moments and friction forces). In the case where all forces are conservative, <u>energy is conserved</u>; i.e., E = T+V = constant; $T_2 + V_2 = T_1 + V_1$

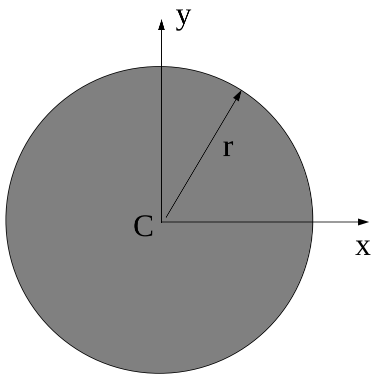
**Inertia Properties:**

Ring



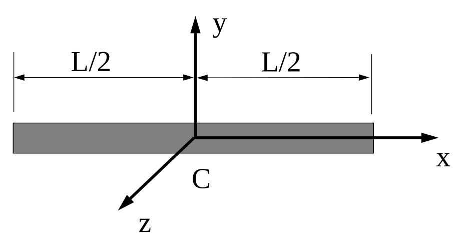

$$I_{xx}^C = I_{yy}^C = \frac{1}{2} m r^2 \quad ; \quad I_{zz}^C \cong m r^2$$

Thin disk



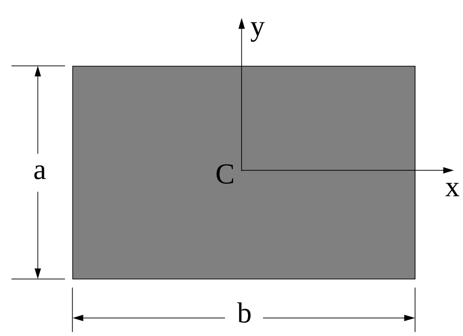

$$I_{xx}^C = I_{yy}^C = \frac{1}{4} m r^2 \quad ; \quad I_{zz}^C \cong \frac{1}{2} m r^2$$

Slender rod




$$I_{xx}^C \cong 0; \quad I_{yy}^C = I_{zz}^C = \frac{1}{12} m L^2$$

Thin rectangular plate




$$I_{xx}^C = \frac{1}{12} m a^2 \; ; \; I_{yy}^C = \frac{1}{12} m b^2$$

$$I_{zz}^C = \frac{1}{12} m \left(a^2 + b^2\right)$$