

Power Generation Cycle Analysis

A Report
Presented to
James G. Hartley, Ph.D.

By:

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In Partial Fulfillment
Of the Requirements for the Course
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Abstract

Purpose:	To evaluate the performance of stationary gas-turbine power plant used to produce electric power. The evaluation will take into account varying ambient temperatures and heat transfer rates in the combustion chamber.																																									
Method:	Use given information about cycle and apply fundamental principals of Thermodynamics to the components of the cycle and the cycle itself in order to obtain a system of equations for the EES program to solve. After solutions have been obtained, analysis of these solution to ascertain their validity was conducted.																																									
Results: <i>Red Italics = Not Possible</i>	Based on ambient temperatures of 0° C, 25°C, 40°C and a constant heat transfer rate of 8,000 kW to the combustion chamber the following data was obtained.																																									
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Conclusions	The initial operating conditions are not optimal for supplying the electric generator on the jobsite with power. Based on the operating conditions that were to be tested, the system operating a 25°C and a heat transfer rate of 10,000 kW will provide the maximum feasible input power to the generator on site. Further investigation found optimal operating condition outside those to be tested. These conditions are 40°C and a 10,000 kW heat transfer rate. These conditions will deliver 2,534 kW to the electric generator.																																									

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Nomenclature

<u>Symbol</u>	<u>Meaning</u>
$\frac{dE_{system}}{dt}$	Change in Total Energy of System With Respect to Time. (kJ/s)
$\frac{ds_{sys}}{dt}$	Change in Entropy of System _{sys} With Respect to Time (kJ/kg-K-s)
EES	Engineering Equation Solver
$h_{Brayton}$	Ideal Brayton Cycle Efficiency
$h_{compressor}$	Adiabatic Compressor Efficiency
h_{cycle}	Cycle Thermal Efficiency
$h_{turbine}$	Adiabatic Turbine Efficiency
h_i	Enthalpy of Air at Indicated State (kJ/kg-K)
$h_{i,s}$	Enthalpy of Air at Indicated Isentropic State (kJ/kg-K)
$\dot{I}_{combustion}$	Irreversibility of Combustion Process (kW)
$\dot{I}_{compressor}$	Irreversibility of Compressor (kW)
\dot{I}_{cycle}	Total Irreversibility of Brayton Cycle (kW)
$\dot{I}_{heatexchanger}$	Irreversibility of Heat Exchanger (kW)
\dot{m}	Mass Flow Rate (kg/s)
P_i	Absolute Pressure at the Corresponding State (kPa)

\dot{Q}_L	Heat Transfer Rate to Heat Exchanger (kW)
\dot{Q}_H	Heat Transfer Rate to Combustion Process (kW)
s_i	Entropy of Air at Indicated State (kJ/kg-K)
T_0	Ambient Temperature (°C)
$T_{reservoir}$	Absolute Temperature of Thermal Energy Reservoir (K)
$\dot{W}_{by,compressor}$	Power Required by the Compressor (kW)
$\dot{W}_{by,generator}$	Power Required by the Electric Generator (kW)
$\dot{W}_{by,turbine}$	Power Produced by the Turbine (kW)
$\dot{W}_{to,generator}$	Power Supplied to the Electric Generator (kW)
$\dot{W}_{turbine}$	Power Produced by the Turbine (kW)

Introduction

A power generation cycle is used to supply power to an electric generator on a job site. A Brayton Cycle is used to supply power to an electric generator. Figure 1 shows the schematic layout, as well as sources of heat transfer and work interactions with the surroundings. Given \dot{Q}_H , relating equations for the compressor and turbine from the manufacturer, and properties of air at state 1 (seen in figure 1), all of the properties in the system were calculated.

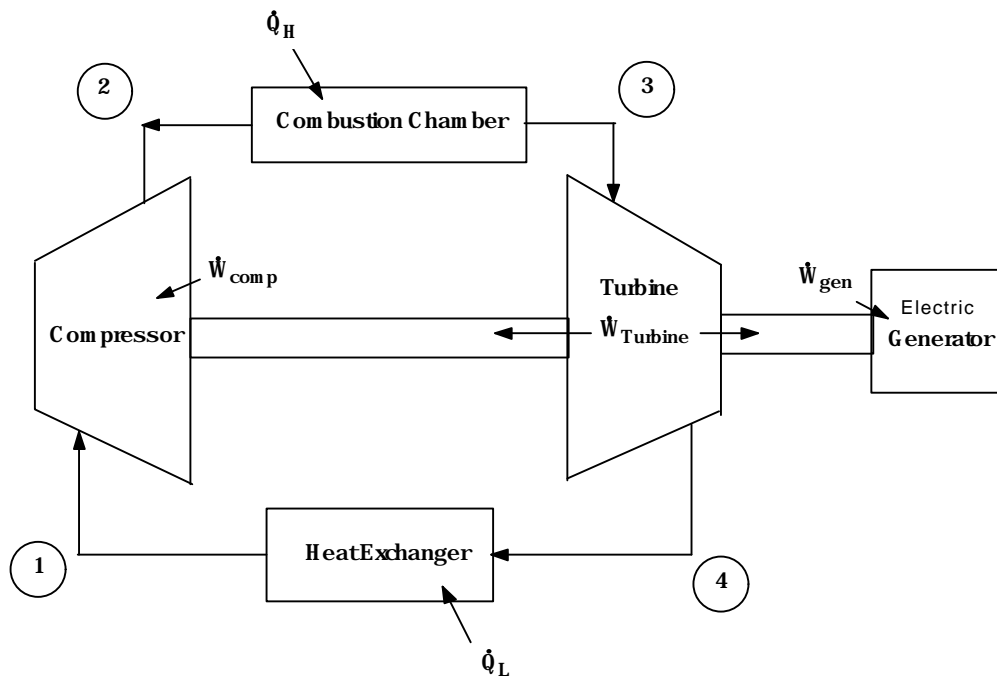


Figure 1: Schematic Layout of Brayton Power Cycle

Challenges were faced in integrating the manufacturer's data on the compressor and turbine into the fundamental thermodynamic equations. In addition, careful attention was used to ensure the manufacturer's data was implemented correctly. Care was also taken to properly label

all values as well as sign conventions for the fundamental thermodynamic equations. This had to be maintained in order to obtain accurate results from the EES (Engineering Equation Solver). A T-s diagram is presented to clarify the irreversibility that was generated by the cycle. In addition, various charts and graphs are offered to summarize and display the data obtained from EES.

Evaluation Overview

Efficiencies

Adiabatic efficiencies were calculated for the turbine, compressor, and the total cycle. Table 1 contains these various efficiencies for an ambient temperature of 0°C and varying heat transfer rates during the combustion process. Notice the efficiency of the turbine is not possible.

Table 1: Cycle Efficiencies with Constant Ambient Temperature (0° C)

T_0 (°C)	\dot{Q}_H (kW)	$h_{turbine}$ (%)	$h_{compressor}$ (%)	h_{cycle} (%)
0	7,000	73.42	87.35	12.79
0	8,000	77.10	85.78	15.74
0	9,000	82.75	82.27	19.22
0	10,000	90.08	77.40	23.04
0	12,000	<i>101.10</i>	71.76	28.17

Table 2 is comprised of efficiencies for variable heat transfer rates for the combustion process with a constant ambient temperature of 25°C.

Table 2: Cycle Efficiencies with Constant Ambient Temperature (25° C)

T_0 (°C)	\dot{Q}_H (kW)	$h_{turbine}$ (%)	$h_{compressor}$ (%)	h_{cycle} (%)
25	7,000	75.32	95.02	14.18
25	8,000	78.46	92.78	17.23
25	9,000	84.57	88.45	20.08
25	10,000	91.86	83.26	24.49
25	12,000	<i>101.80</i>	78.05	29.05

Table 3 is composed of efficiencies for a constant ambient temperature of 40°C and various heat transfer rates during the combustion process.

Table 3: Cycle Efficiencies with Constant Ambient Temperature (40° C)

T_0 (°C)	\dot{Q}_H (kW)	$h_{turbine}$ (%)	$h_{compressor}$ (%)	h_{cycle} (%)
40	7,000	74.92	99.52	15.05
40	8,000	79.32	96.83	18.15
40	9,000	85.68	92.02	21.77
40	10,000	92.86	86.74	25.34
40	12,000	102.3	81.82	29.57

Tables 1 through 3 demonstrate that cycle efficiencies as well as turbine efficiencies are directly proportional to heat transfer rates. They also show the compressor efficiencies are inversely proportional to heat transfer rates. The data for a heat transfer rate of 12,000 kW during the combustion process is unreasonable. An efficiency of over 100% with the 12,000 kW in combustion results from this unreasonable heat generation. Thus there is a limit as to how high the heat transfer rate can be. This data also says that at an operating condition of 40°C ambient temperature and a heat transfer rate of 10,000 kW the operating conditions will lead to the most efficient use of resources. However, it may be difficult to supply the system with an ambient temperature of 40 degrees Celsius.

Power Supplied to Electric Generator

Table 4 shows power supplied to the generator with varying ambient temperatures and heat transfer rates. Close examination of the data shows that the operating conditions, which will lead to the maximum power being delivered to the generator, are 40°C ambient temperature and a heat transfer rate of 10,000 kW. These conditions also happen to be the conditions at which the maximum efficiencies occur. Again the magnitude of a 40 degree ambient temperature is difficult to achieve in practice.

Table 4: Power Supplied to Electric Generator

T_0 (°C)	\dot{Q}_H (kW)	$\dot{W}_{to,generator}$ (kW)
0	7,000	895.3
0	8,000	1,259
0	9,000	1,729
0	10,000	2,304
0	12,000	3,380
25	7,000	992.7
25	8,000	1,378
25	9,000	1,872
25	10,000	2,449
25	12,000	3,487
40	7,000	1,053
40	8,000	1,452
40	9,000	1,959
40	10,000	2,534
40	12,000	3,549

Mass Flowrate and Pressure Exiting the Compressor

The mass flowrate of air circulating through the systems and as the absolute pressure of air leaving the compressor were calculated for the same ambient temperatures and heat transfer rates as used in calculating efficiencies and power generation. Table 5 presents the results of these calculations. Per the conservation of mass, every process in the cycle has the same mass flowrate (see Methods section). From information provided by the compressor manufacturer, power required to drive the compressor is proportional to compressor exit pressure. Compressor exit pressure is inversely proportional to mass flowrate. Inspection of data in Table 5 agrees with the information provided by the manufacturer.

Table 5: Mass Flowrates and Compressor Exit Pressure

T_0 (°C)	\dot{Q}_H (kW)	\dot{m} (kg/sec)	P_2 (kPa)
0	7,000	11.74	310.7
0	8,000	11.24	334.6
0	9,000	10.59	362.0
0	10,000	9.89	387.7
0	12,000	7.17	410.3
25	7,000	11.61	317.3
25	8,000	11.07	341.9
25	9,000	10.4	369.3
25	10,000	9.75	392.6
25	12,000	9.14	411.1
40	7,000	11.52	321.3
40	8,000	10.97	346.4
40	9,000	10.29	373.5
40	10,000	9.67	395.2
40	12,000	9.13	411.5

Irreversibility

Irreversibility rates for every process of the cycle and for the total cycle were calculated and tabulated in Table 6. All irreversibilities seem to be reasonable except for the irreversibility of the turbine at 12,000 kW. This negative value for irreversibility stems from the unrealistic heat generation, which is discussed in the Introduction section.

Table 6: Irreversibility Rates for Components and Cycle

T_0 (°C)	\dot{Q}_H (kW)	$\dot{I}_{combustion}$ (kW)	$\dot{I}_{compressor}$ (kW)	$\dot{I}_{heatexchanger}$ (kW)	$\dot{I}_{turbine}$ (kW)	\dot{I}_{cycle} (kW)
0	7,000	1959	137.2	2348	333.8	4779
0	8,000	1955	158.0	2816	296.5	5225
0	9,000	1873	202.3	3262	228.0	5566
0	10,000	1734	264.1	3672	131.5	5801
0	12,000	1548	335.7	4477	-14.11	6346
25	7,000	1807	50.49	2498	325.6	4681
25	8,000	1795	75.11	2955	280.9	5106
25	9,000	1710	123.6	3385	204.4	5423
25	10,000	1586	183.1	3779	107.7	5656
25	12,000	1436	243.6	4585	-24.23	6240
40	7,000	1722	4.66	2575	319.8	4621
40	8,000	1705	31.79	3025	270.6	5032
40	9,000	1620	82.34	3444	189.8	5336
40	10,000	1506	139.6	3832	94.4	5572
40	12,000	1373	194.0	4641	-29.8	6178

Figure 2 shows a plot of Temperature vs. Entropy (T-s diagram) for a Brayton cycle. The dashed vertical lines represent constant entropy, adiabatic processes (isentropic), denoted with a subscript s. The solid lines represent the path of the actual cycle. The results coincide with theory because losses occur from state 1 to state 2 and from state 3 to state 4. Energy losses are seen in the efficiencies for the cycle located in tables 1 through 3.

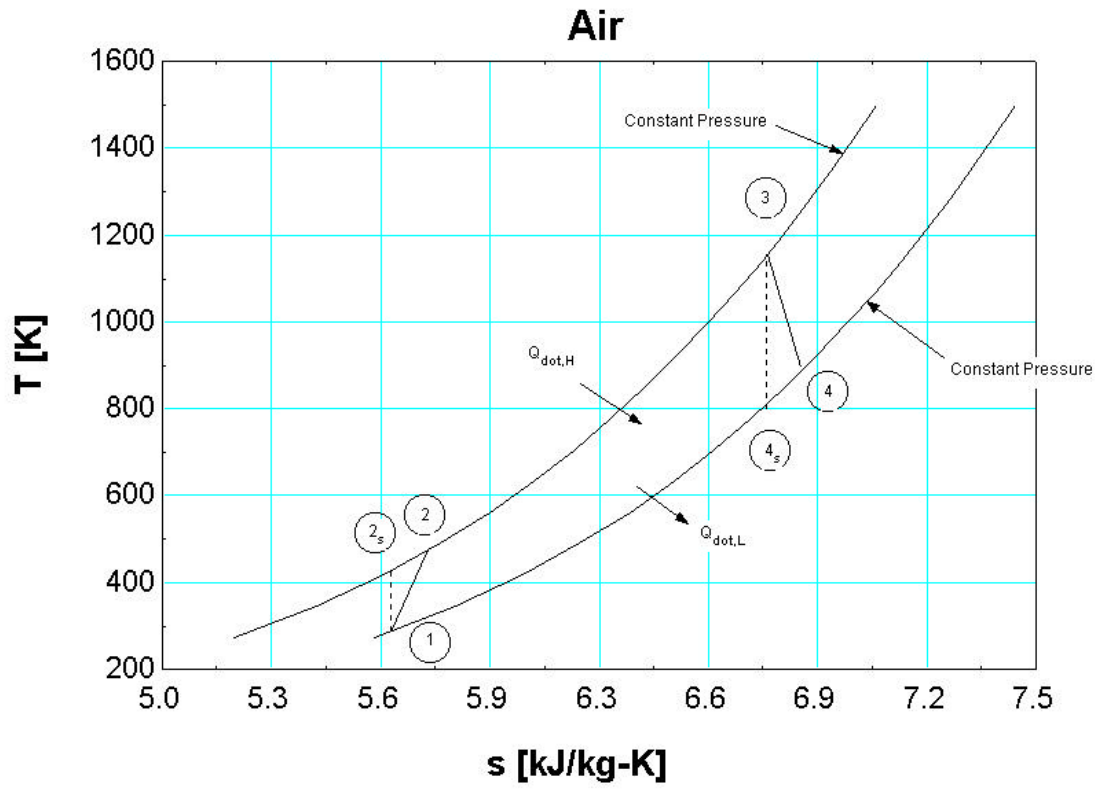


Figure 2: T-s Diagram for Brayton Cycle

Methods

Assumptions

The following assumptions were made to simplify the problem and allow calculations to be made.

1. System is an Ideal Brayton Cycle.
2. Air Standard Assumptions
 - a. The working fluid has properties that are the same as those of air.
 - b. The working fluid is an ideal gas
 - c. Heat transfer from the cycle to a low-temperature reservoir replaces the exhaust process and heat transfer to the cycle from a high-temperature reservoir replaces the combustion process.
3. The system operates at steady state.
4. Uniform flow throughout the system.
5. Combustion temperature is 1,300°C (1573°K)
6. There are negligible kinetic and potential energy changes.
7. Negligible pressure drop across combustion and exchanger.

Manufacturer's Data

The following performance data was provided by the manufacturer.

Compressor

$$P_2 = 331 + 45.6 \dot{m} - 4.03 \dot{m}^2 \quad (1)$$

$$W_{comp} = 1020 - 0.383 P_2 + 0.00513 P_2^2 \quad (2)$$

Where P_2 = discharge pressure of compressor, kPa; \dot{m} = mass flow rate, kg/s; \dot{W}_{comp} = power required by the compressor, kW. Figure 3 is a plot of compressor discharge pressure vs. mass flowrate. It shows that discharge pressure is inversely proportional to mass flowrate.

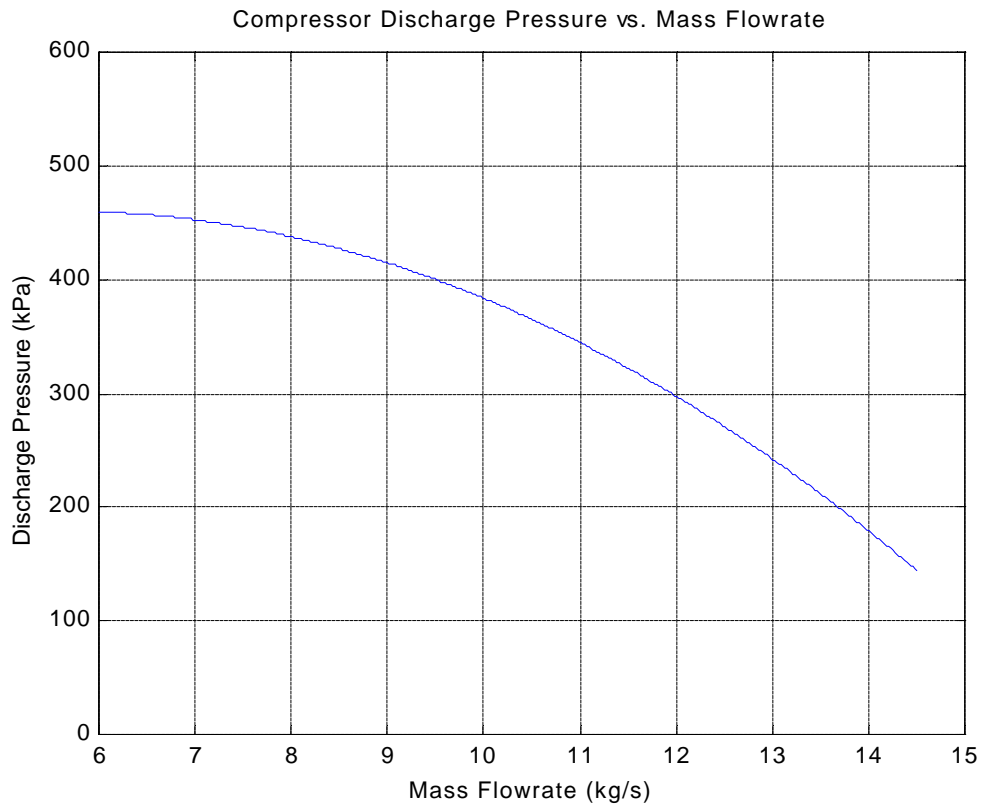


Figure 3: Compressor Discharge Pressure vs. Mass Flowrate

Figure 4 is a plot of power required by the compressor vs. discharge pressure. It shows a direct proportionality between the two quantities.

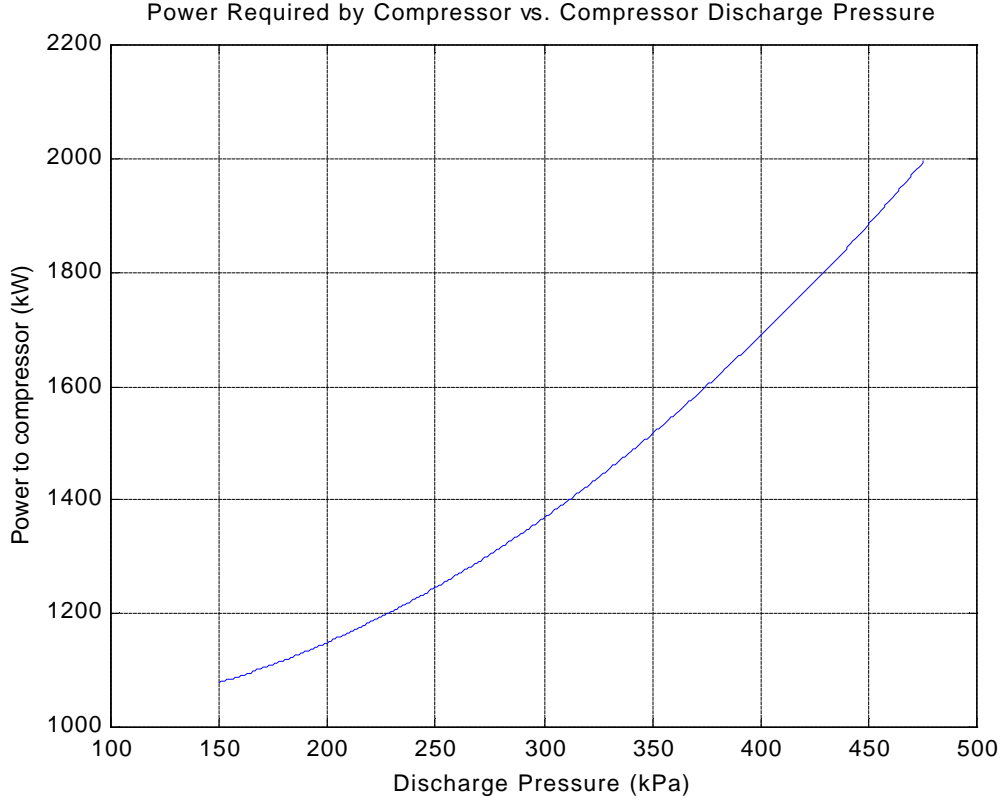


Figure 4: Power Required to Operate Compressor vs. Compressor Discharge Pressure

Turbine

$$\dot{m} = 8.5019 + 0.02332P_3 + 0.48 \times 10^{-4} P_3^2 - 0.2644T_3 + 0.1849 \times 10^{-4} T_3^2 + 0.000121P_3T_3 - 0.2736 \times 10^{-6} P_3^2T_3 - 0.1137 \times 10^{-6} P_3 T_3^2 + 0.2124 \times 10^{-9} P_3^2T_3^2 \quad (3)$$

$$\dot{W}_{turbine} = 1727.5 - 10.06P_3 + 0.33033P_3^2 - 7.4709T_3 + 0.003919T_3^2 + 0.05092P_3T_3 - 0.8525 \times 10^{-4} P_3^2T_3 - 0.2356 \times 10^{-4} P_3 T_3^2 + 0.4473 \times 10^{-7} P_3^2T_3^2 \quad (4)$$

Where P_3 = inlet pressure of turbine, kPa; \dot{m} = mass flow rate, kg/s; $\dot{W}_{turbine}$ = power output by

the turbine, kW; T_3 = inlet temperature to turbine, °C. The manufacturer's equations are not consistent with units because the units are considered in the coefficients of the variables. Figure 5 is a plot of power produced by the turbine vs. turbine inlet pressure for various inlet temperatures. It shows that inlet pressure is directly proportional to mass flowrate.

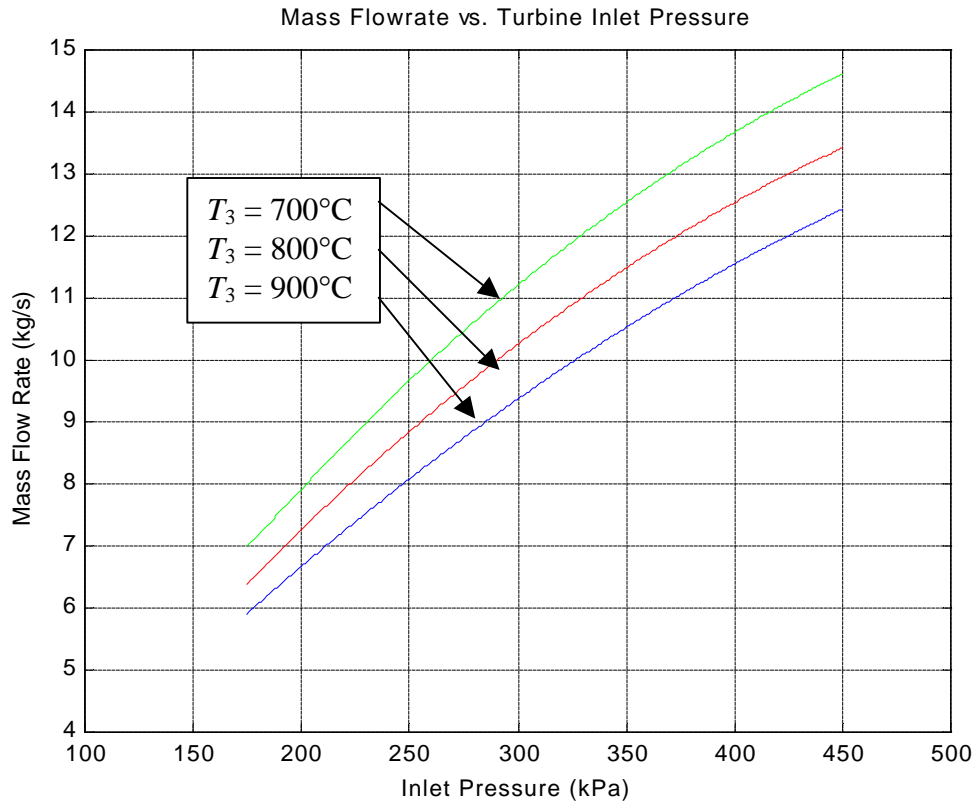


Figure 5: Mass Flowrate vs. Turbine Inlet Pressure

Figure 6 is a plot of power produced by the turbine vs. turbine inlet pressure for various temperatures. It shows a direct proportionality between the two quantities.

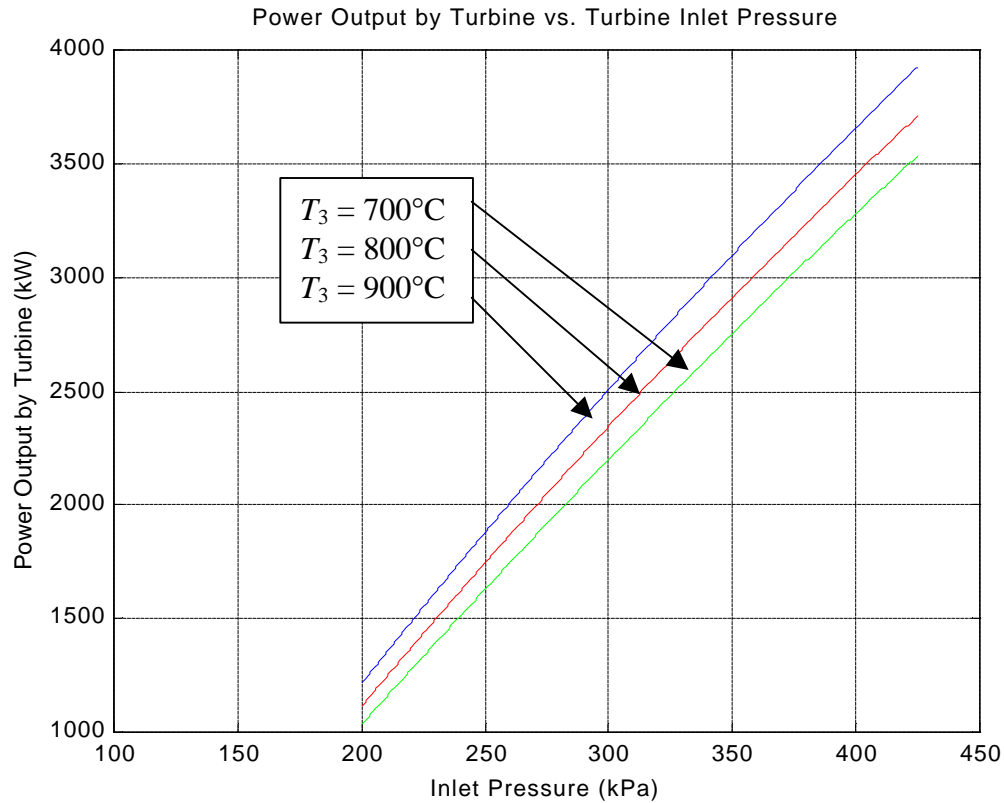


Figure 6: Power Produced by the Turbine vs. Inlet Pressure

The preceding plots were generated using Matlab with the performance equations provided by the manufacturer. They were used to further confirm the validity of computations.

Conservation of Mass

Based on the assumption of uniform flow and steady state operation, conservation of mass equations for every device in the cycle can be simplified from integral form and written as follows:

$$\dot{m}_{inlet} = \dot{m}_{exit} \quad (5)$$

This leads to the conclusion that the mass flowrate of one device is the same as the mass flowrate for every other device, which is seen in equation 6.

$$\dot{m}_{compressor} = \dot{m}_{combustion} = \dot{m}_{turbine} = \dot{m}_{heatexchanger} \equiv \dot{m} \quad (6)$$

Conservation of Energy

Adiabatic Processes

The compressor and turbine are assumed to have negligible potential and kinetic energy changes. The general conservation of energy equations for these processes can be written as follows:

$$\dot{Q}_{to,sys} - \dot{W}_{by,sys} + \dot{m}[h_{inlet} - h_{exit}] = \frac{dE}{dt} \quad (7)$$

Using the assumptions, this equation is simplified to:

$$\dot{W}_{by,sys} = \dot{m}[h_{exit} - h_{inlet}] \quad (8)$$

Non-adiabatic Processes

The combustion and heat exchanger processes are not adiabatic; the general conservation of energy equation for these processes is the same as equation (7). Assuming there is not work interaction with the surroundings equation (7) can be simplified to the following:

$$\dot{Q}_{to,sys} = \dot{m}[h_{inlet} - h_{exit}] \quad (9)$$

Electric Generator

The power supplied to the electric generator on the job-site is delivered from the turbine, but not all the power from the turbine goes to the generator. Some of the turbine power goes to operate the compressor. The relationship between these three quantities is:

$$\dot{W}_{by,turbine} = \left| \dot{W}_{by,generator} \right| + \left| \dot{W}_{by,compressor} \right| \quad (10)$$

Efficiencies

Adiabatic turbine and compressor efficiency equations were obtained from *Thermodynamics, Third Edition* (Black and Hartley). They are as follows:

$$\mathbf{h}_{turbin} = \frac{h_{inlet} - h_{exit}}{h_{inlet} - h_{exit,s}} \quad (11)$$

$$\mathbf{h}_{compressor} = \frac{h_{exit,s} - h_{inlet}}{h_{exit} - h_{inlet}} \quad (12)$$

The thermal efficiency for an Ideal Brayton Cycle, also obtained from Black and Hartley is:

$$\mathbf{h}_{Brayton} = 1 + \frac{h_{inlet,compressor} - h_{exit,turbine}}{h_{inlet,turbine} - h_{exit,compressor}} \quad (13)$$

or,

$$\mathbf{h}_{Brayton} = 1 + \frac{\dot{Q}_L}{\dot{Q}_H} \quad (14)$$

Irreversibility

The general equation for irreversibility is as follows:

$$\dot{I} = T_0 \left[\dot{m} \left(\frac{ds_{sys}}{dt} \right) + \sum_{i=0}^n \frac{\dot{Q}_i}{T_i} \right] \quad (15)$$

Adiabatic Processes

For the Adiabatic Process the heat transfer term in equation (15) is zero and the irreversibility equation reduces to:

$$\dot{I} = T_0 \left[\dot{m} (s_{exit} - s_{inlet}) \right] \quad (16)$$

Non-Adiabatic Processes

For the non-adiabatic process no terms are zero in equation (15). However, there is only one thermal energy reservoir and the derivative term can be integrated to obtain an irreversibility equation for the combustion and heat exchanger processes.

$$\dot{I} = T_0 \left[\dot{m}(s_{exit} - s_{inlet}) + \frac{\dot{Q}_{to, reservoir}}{T_{reservoir}} \right] \quad (17)$$

Cycle Irreversibility

The total Irreversibility for the cycle is just the sum of the processes for the cycle:

$$\dot{I}_{cycle} = \dot{I}_{compressor} + \dot{I}_{combustion} + \dot{I}_{turbine} + \dot{I}_{heatexchanger} \quad (18)$$

Engineering Equation Solver (EES) Method

The equations provide by the turbine and compressor manufacturer, the conservation of energy equations, the efficiency and irreversibility equations were coded into EES along with the given initial conditions. A complete copy of the EES code is presented in appendix A.

Tables 7 and 8 show the parametric tables created to solve the system of equations obtained. Values in blue are the dependent variables; values in black are the independent variables. Table 7 solved for mass flow rate, work to the generator, pressures, and efficiencies of all of the components in the system. Table 8 was used to solve for the irreversibility of the compressor, turbine, combustion process, and heat exchanger. In addition, Table 8 solved for total irreversibility for the cycle.

Table 7: Parametric Table of Power to Electric Generator, Mass Flowrate, Efficiencies, and Compressor Exit Pressures

T_0 (°C)	\dot{Q}_H (kW)	\dot{m} (kg/sec)	$\dot{W}_{to,generator}$ (kW)	P_2 (kPa)	$h_{turbine}$ (%)	$h_{compressor}$ (%)	h_{cycle} (%)
0	7,000	11.74	895.3	310.7	73.42	87.35	12.79
0	8,000	11.24	1,259	334.6	77.10	85.78	15.74
0	9,000	10.59	1,729	362.0	82.75	82.27	19.22
0	10,000	9.89	2,304	387.7	90.08	77.40	23.04
0	12,000	7.17	3,380	410.3	101.10	71.76	28.17
25	7,000	11.61	992.7	317.3	75.32	95.02	14.18
25	8,000	11.07	1,378	341.9	78.46	92.78	17.23
25	9,000	10.4	1,872	369.3	84.57	88.45	20.08
25	10,000	9.75	2,449	392.6	91.86	83.26	24.49
25	12,000	9.14	3,487	411.1	101.80	78.05	29.05
40	7,000	11.52	1,053	321.3	74.92	99.52	15.05
40	8,000	10.97	1,452	346.4	79.32	96.83	18.15
40	9,000	10.29	1,959	373.5	85.68	92.02	21.77
40	10,000	9.67	2,534	395.2	92.86	86.74	25.34
40	12,000	9.13	3,549	411.5	102.3	81.82	29.57

Table 8: Parametric Table of Irreversibilities

T_0 (°C)	\dot{Q}_H (kW)	$\dot{I}_{combustion}$ (kW)	$\dot{I}_{compressor}$ (kW)	$\dot{I}_{heatexchanger}$ (kW)	$\dot{I}_{turbine}$ (kW)	\dot{I}_{cycle} (kW)
0	7,000	1,959	137.2	2,348	333.8	4,779
0	8,000	1,955	158.0	2,816	296.5	5,225
0	9,000	1,873	202.3	3,262	228.0	5,566
0	10,000	1,734	264.1	3,672	131.5	5,801
0	12,000	1,548	335.7	4,477	-14.11	6,346
25	7,000	1,807	50.49	2,498	325.6	4,681
25	8,000	1,795	75.11	2,955	280.9	5,106
25	9,000	1,710	123.6	3,385	204.4	5,423
25	10,000	1,586	183.1	3,779	107.7	5,656
25	12,000	1,436	243.6	4,585	-24.23	6,240
40	7,000	1,722	4.66	2,575	319.8	4,621
40	8,000	1,705	31.79	3,025	270.6	5,032
40	9,000	1,620	82.34	3,444	189.8	5,336
40	10,000	1,506	139.6	3,832	94.4	5,572
40	12,000	1,373	194.0	4,641	-29.8	6,178

Conclusion

The values calculated behaved as expected based on theory from thermodynamics. As the heat generation increased, the efficiencies for the turbine and system increased, and the efficiency for the compressor decreased. All of the efficiencies increased as temperature

increased. Irreversibility decreases as the temperature increases, but decreases as the heat transfer rate increases. This phenomenon is due to the dependence on entropy. The difference in values for entropy increases as temperature and heat transfer increase. In closing, the calculated values were acceptable with reasonable input variables.

Appendix A

Engineering Equation Solver Code

*"!Group 4"
"!ME 4315"
"!Dr. Hartley"*

"State One"

$P_1 = 101$ *{Given In Problem}*

*"T_1 = 25" {Given In Problem. Declared in Parametric Table. Shown here for completeness.
Commented out so not used in calculations.}*

$h_1 = \text{enthalpy}(\text{AIR}, T=T_1)$

$s_1 = \text{entropy}(\text{AIR}, T=T_1, P=P_1)$

"State Two"

$P_2 = P_3$ *{From Process Diagram}*

$h_2 = \text{enthalpy}(\text{AIR}, T=T_2)$

$s_2 = \text{entropy}(\text{AIR}, T=T_2, P=P_2)$

"State Two Isentropic"

$s_{2_s} = s_1$

$s_{2_s} = \text{entropy}(\text{AIR}, T=T_{2_s}, P=P_2)$

$h_{2_s} = \text{enthalpy}(\text{AIR}, T=T_{2_s})$

"State Three"

$h_3 = \text{enthalpy}(\text{AIR}, T=T_3)$

$s_3 = \text{entropy}(\text{AIR}, T=T_3, P=P_3)$

"State Four"

$P_4 = P_1$

$h_4 = \text{enthalpy}(\text{AIR}, T=T_4)$

$s_4 = \text{entropy}(\text{AIR}, T=T_4, P=P_4)$

"State Four Isentropic"

$$h_{4_s} = \text{enthalpy}(\text{AIR}, T=T_{4_s})$$

$$s_{4_s} = s_3$$

$$s_{4_s} = \text{entropy}(\text{AIR}, T=T_{4_s}, P=P_4)$$

"!From Compressor and Turbine data sheet from Dr. Hartley"

"Compressor"

$$P_2 = 331 + 45.6 * m_{\text{dot}} - 4.03 * m_{\text{dot}}^2 \quad \{\text{Given In Problem}\}$$

$$w_{\text{dot}_c_{\text{req}}} = 1020 - 0.383 * P_2 + 0.00513 * P_2^2 \quad \{\text{Given In Problem}\}$$

"Turbine"

$$m_{\text{dot}} = 8.5019 + 0.02332 * P_3 + 0.000048 * P_3^2 - 0.02644 * T_3 + 0.00001849 * T_3^2 + 0.000121 * P_3 * T_3 - 2.736E-7 * P_3^2 * T_3 - 1.137E-7 * P_3 * T_3^2 + 2.124E-10 * P_3^2 * T_3^2$$

{Given In Problem}

$$(w_{\text{dot}_{\text{turbine}}}) = 1727.5 - 10.06 * P_3 + 0.033033 * P_3^2 - 7.4709 * T_3 + 0.003919 * T_3^2 + 0.050921 * P_3 * T_3 - 0.00008525 * P_3^2 * T_3 - 0.00002356 * P_3 * T_3^2 + 4.473E-8 * P_3^2 * T_3^2$$

{Given In Problem}

"!Conservation of Energy Equations"

"Compressor"

$$w_{\text{dot}_c} = m_{\text{dot}} * (h_1 - h_2)$$

$$w_{\text{dot}_c} = -w_{\text{dot}_c_{\text{req}}}$$

"Combustion Chamber"

$$Q_{\text{dot}_H} = m_{\text{dot}} * (h_3 - h_2)$$

"Q_dot_H = 8000" {Given In Problem. Declared in Parametric Table. Shown here for completeness. Commented out so not used in calculations.}

"Turbine"

$$w_{\text{dot}_{\text{turbine}}} = m_{\text{dot}} * (h_3 - h_4)$$

$$w_{\text{dot}_{\text{turbine}}} = w_{\text{dot}_c_{\text{req}}} + w_{\text{dot}_{\text{gen}}}$$

"Heat Exchanger"

$$Q_{\text{dot}_L} = m_{\text{dot}} * (h_1 - h_4)$$

"!Efficiencies"

"Adiabatic Turbine"

$$\eta_{\text{turbine}} = (h_3 - h_4) / (h_3 - h_{4_s})$$

"Adiabatic Compressor"

$$\eta_{\text{compressor}} = (h_{2_s} - h_1) / (h_2 - h_1)$$

"Brayton Cycle"

$$\eta_{\text{cycle}} = 1 + ((h_1 - h_4) / (h_3 - h_2))$$

"eta_c = 1 + Q_dot_L / Q_dot_H"

"!Irreversibility Rates"

$$T_0 = 25 + 273 \quad \{Given\ In\ Problem\}$$

$$T_{\text{combustion}} = 1300 + 273 \quad \{Given\ In\ Problem.\}$$

"Compressor"

$$I_{\text{dot}_Compressor} = T_0 * m_{\text{dot}} * (s_2 - s_1)$$

"Combustion Chamber"

$$I_{\text{dot}_Combustion} = T_0 * (m_{\text{dot}} * (s_3 - s_2) + (-Q_{\text{dot}_H} / T_{\text{combustion}}))$$

"Turbine"

$$I_{\text{dot}_Turbine} = T_0 * m_{\text{dot}} * (s_4 - s_3)$$

"Heat Exchanger"

$$I_{\text{dot}_HeatExchanger} = T_0 * (m_{\text{dot}} * (s_1 - s_4) + (-Q_{\text{dot}_L} / T_0))$$

"Cycle"

$$I_{\text{dot}_cycle} = I_{\text{dot}_Compressor} + I_{\text{dot}_Combustion} + I_{\text{dot}_Turbine} + I_{\text{dot}_HeatExchanger}$$

Appendix B

Matlab Code for Performance Plots

Compressor Discharge Pressure vs. Mass Flowrate

```
close all
```

```
P = [0: 1/1000: 14.5];
```

```
f = [331 + 45.6 .* P - 4.03 .* P.^2];
```

```
figure(1)
```

```
plot(P,f)
```

```
grid on
```

```
axis([6 15 0 600])
```

```
xlabel('Mass Flowrate (kg/s)')
```

```
ylabel('Discharge Pressure (kPa)')
```

```
hold on
```

```
title(strcat('Compressor Discharge Pressure vs. Mass Flowrate'))
```

Power Required by Compressor vs. Compressor Discharge Pressure

```
close all

P = [150: 1/1000: 475];
f = [1020 - 0.383 .* P + 0.00513 .* P.^2];

figure(1)
plot(P,f)
grid on
axis([100 500 1000 2200])
xlabel('Discharge Pressure (kPa)')
ylabel('Power to compressor (kW)')

hold on

title(strcat('Power Required by Compressor vs. Compressor Discharge Pressure'))
```

Mass Flowrate vs. Turbine Inlet Pressure

```
close all

P = [175: 1/1000: 450];
T = [700];
f = [8.5019 + 0.02332 .* P + 0.000048 .* P.^2 - 0.02644 .* T + 0.00001849 .*
T.^2 + 0.000121 .* P .* T - 2.736e-7 .* P.^2 .* T - 1.137e-7 .* P .* T.^2
+ 2.124e-10 .* P.^2 .* T.^2];

figure(1)
plot(P,f,'g')
grid on
axis([100 500 4 15])
xlabel('Inlet Pressure (kPa)')
ylabel('Mass Flow Rate (kg/s)')

T =[800];
f = [8.5019 + 0.02332 .* P + 0.000048 .* P.^2 - 0.02644 .* T + 0.00001849 .*
T.^2 + 0.000121 .* P .* T - 2.736e-7 .* P.^2 .* T - 1.137e-7 .* P .* T.^2
+ 2.124e-10 .* P.^2 .* T.^2];

hold on
plot(P,f,'r')

T =[900];
f = [8.5019 + 0.02332 .* P + 0.000048 .* P.^2 - 0.02644 .* T + 0.00001849 .*
T.^2 + 0.000121 .* P .* T - 2.736e-7 .* P.^2 .* T - 1.137e-7 .* P .* T.^2
+ 2.124e-10 .* P.^2 .* T.^2];

hold on
plot(P,f)

hold on
title(strcat('Mass Flowrate vs. Turbine Inlet Pressure'))
```

Power Output by Turbine vs. Turbine Inlet Pressure

```
close all

P = [200: 1/1000: 425];
T = [700];
f = [1727.5 - 10.06 .* P + 0.033033 .* P.^2 - 7.4709 .* T + 0.003919 .* T
.^2 + 0.050921 .* P .* T - 0.8525e-4 .* P.^2 .* T - 0.2356e-4 .* P .* T.^2
+ 0.4473e-7 .* P.^2 .* T.^2];

figure(1)
plot(P,f,'g')
grid on
axis([100 450 1000 4000])
xlabel('Inlet Pressure (kPa)')
ylabel('Power Output by Turbine (kW)')

T =[800];
f = [1727.5 - 10.06 .* P + 0.033033 .* P.^2 - 7.4709 .* T + 0.003919 .* T
.^2 + 0.050921 .* P .* T - 0.8525e-4 .* P.^2 .* T - 0.2356e-4 .* P .* T.^2
+ 0.4473e-7 .* P.^2 .* T.^2];

hold on
plot(P,f,'r')

T =[900];
f = [1727.5 - 10.06 .* P + 0.033033 .* P.^2 - 7.4709 .* T + 0.003919 .* T
.^2 + 0.050921 .* P .* T - 0.8525e-4 .* P.^2 .* T - 0.2356e-4 .* P .* T.^2
+ 0.4473e-7 .* P.^2 .* T.^2];hold on

hold on
plot(P,f)

hold on
title(strcat('Power Output by Turbine vs. Turbine Inlet Pressure'))
```