



Introduction to System Dynamics

1-1 INTRODUCTION

System dynamics deals with the mathematical modeling of dynamic systems and response analyses of such systems with a view toward understanding the dynamic nature of each system and improving the system's performance. Response analyses are frequently made through computer simulations of dynamic systems.

Because many physical systems involve various types of components, a wide variety of different types of dynamic systems will be examined in this book. The analysis and design methods presented can be applied to mechanical, electrical, pneumatic, and hydraulic systems, as well as nonengineering systems, such as economic systems and biological systems. It is important that the mechanical engineering student be able to determine dynamic responses of such systems.

We shall begin this chapter by defining several terms that must be understood in discussing system dynamics.

Systems. A *system* is a combination of components acting together to perform a specific objective. A *component* is a single functioning unit of a system. By no means limited to the realm of the physical phenomena, the concept of a system can be extended to abstract dynamic phenomena, such as those encountered in economics, transportation, population growth, and biology.

A system is called *dynamic* if its present output depends on past input; if its current output depends only on current input, the system is known as *static*. The output of a static system remains constant if the input does not change. The output changes only when the input changes. In a dynamic system, the output changes with time if the system is not in a state of equilibrium. In this book, we are concerned mostly with dynamic systems.

Mathematical models. Any attempt to design a system must begin with a prediction of its performance before the system itself can be designed in detail or actually built. Such prediction is based on a mathematical description of the system's dynamic characteristics. This mathematical description is called a *mathematical model*. For many physical systems, useful mathematical models are described in terms of differential equations.

Linear and nonlinear differential equations. Linear differential equations may be classified as linear, time-invariant differential equations and linear, time-varying differential equations.

A *linear, time-invariant differential equation* is an equation in which a dependent variable and its derivatives appear as linear combinations. An example of such an equation is

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 10x = 0$$

Since the coefficients of all terms are constant, a linear, time-invariant differential equation is also called a *linear, constant-coefficient differential equation*.

In the case of a *linear, time-varying differential equation*, the dependent variable and its derivatives appear as linear combinations, but a coefficient or coefficients of terms may involve the independent variable. An example of this type of differential equation is

$$\frac{d^2x}{dt^2} + (1 - \cos 2t)x = 0$$

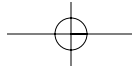
It is important to remember that, in order to be linear, the equation must contain no powers or other functions or products of the dependent variables or its derivatives.

A differential equation is called *nonlinear* if it is not linear. Two examples of nonlinear differential equations are

$$\frac{d^2x}{dt^2} + (x^2 - 1)\frac{dx}{dt} + x = 0$$

and

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + x + x^3 = \sin \omega t$$



Linear systems and nonlinear systems. For linear systems, the equations that constitute the model are linear. In this book, we shall deal mostly with linear systems that can be represented by linear, time-invariant ordinary differential equations.

The most important property of linear systems is that the *principle of superposition* is applicable. This principle states that the response produced by simultaneous applications of two different forcing functions or inputs is the sum of two individual responses. Consequently, for linear systems, the response to several inputs can be calculated by dealing with one input at a time and then adding the results. As a result of superposition, complicated solutions to linear differential equations can be derived as a sum of simple solutions.

In an experimental investigation of a dynamic system, if cause and effect are proportional, thereby implying that the principle of superposition holds, the system can be considered linear.

Although physical relationships are often represented by linear equations, in many instances the actual relationships may not be quite linear. In fact, a careful study of physical systems reveals that so-called linear systems are actually linear only within limited operating ranges. For instance, many hydraulic systems and pneumatic systems involve nonlinear relationships among their variables, but they are frequently represented by linear equations within limited operating ranges.

For nonlinear systems, the most important characteristic is that the principle of superposition is *not* applicable. In general, procedures for finding the solutions of problems involving such systems are extremely complicated. Because of the mathematical difficulty involved, it is frequently necessary to linearize a nonlinear system near the operating condition. Once a nonlinear system is approximated by a linear mathematical model, a number of linear techniques may be used for analysis and design purposes.

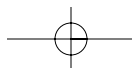
Continuous-time systems and discrete-time systems. Continuous-time systems are systems in which the signals involved are continuous in time. These systems may be described by differential equations.

Discrete-time systems are systems in which one or more variables can change only at discrete instants of time. (These instants may specify the times at which some physical measurement is performed or the times at which the memory of a digital computer is read out.) Discrete-time systems that involve digital signals and, possibly, continuous-time signals as well may be described by difference equations after the appropriate discretization of the continuous-time signals.

The materials presented in this text apply to continuous-time systems; discrete-time systems are not discussed.

1-2 MATHEMATICAL MODELING OF DYNAMIC SYSTEMS

Mathematical modeling. Mathematical modeling involves descriptions of important system characteristics by sets of equations. By applying physical laws to a specific system, it may be possible to develop a mathematical model that describes the dynamics of the system. Such a model may include unknown parameters, which



must then be evaluated through actual tests. Sometimes, however, the physical laws governing the behavior of a system are not completely defined, and formulating a mathematical model may be impossible. If so, an experimental modeling process can be used. In this process, the system is subjected to a set of known inputs, and its outputs are measured. Then a mathematical model is derived from the input–output relationships obtained.

Simplicity of mathematical model versus accuracy of results of analysis.

In attempting to build a mathematical model, a compromise must be made between the simplicity of the model and the accuracy of the results of the analysis. It is important to note that the results obtained from the analysis are valid only to the extent that the model approximates a given physical system.

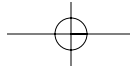
In determining a reasonably simplified model, we must decide which physical variables and relationships are negligible and which are crucial to the accuracy of the model. To obtain a model in the form of linear differential equations, any distributed parameters and nonlinearities that may be present in the physical system must be ignored. If the effects that these ignored properties have on the response are small, then the results of the analysis of a mathematical model and the results of the experimental study of the physical system will be in good agreement. Whether any particular features are important may be obvious in some cases, but may, in other instances, require physical insight and intuition. Experience is an important factor in this connection.

Usually, in solving a new problem, it is desirable first to build a simplified model to obtain a general idea about the solution. Afterward, a more detailed mathematical model can be built and used for a more complete analysis.

Remarks on mathematical models. The engineer must always keep in mind that the model he or she is analyzing is an approximate mathematical description of the physical system; it is not the physical system itself. In reality, no mathematical model can represent any physical component or system precisely. Approximations and assumptions are always involved. Such approximations and assumptions restrict the range of validity of the mathematical model. (The degree of approximation can be determined only by experiments.) So, in making a prediction about a system's performance, any approximations and assumptions involved in the model must be kept in mind.

Mathematical modeling procedure. The procedure for obtaining a mathematical model for a system can be summarized as follows:

1. Draw a schematic diagram of the system, and define variables.
2. Using physical laws, write equations for each component, combine them according to the system diagram, and obtain a mathematical model.
3. To verify the validity of the model, its predicted performance, obtained by solving the equations of the model, is compared with experimental results. (The question of the validity of any mathematical model can be answered only by experiment.) If the experimental results deviate from the prediction



to a great extent, the model must be modified. A new model is then derived and a new prediction compared with experimental results. The process is repeated until satisfactory agreement is obtained between the predictions and the experimental results.

1-3 ANALYSIS AND DESIGN OF DYNAMIC SYSTEMS

This section briefly explains what is involved in the analysis and design of dynamic systems.

Analysis. *System analysis* means the investigation, under specified conditions, of the performance of a system whose mathematical model is known.

The first step in analyzing a dynamic system is to derive its mathematical model. Since any system is made up of components, analysis must start by developing a mathematical model for each component and combining all the models in order to build a model of the complete system. Once the latter model is obtained, the analysis may be formulated in such a way that system parameters in the model are varied to produce a number of solutions. The engineer then compares these solutions and interprets and applies the results of his or her analysis to the basic task.

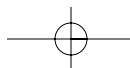
It should always be remembered that deriving a reasonable model for the complete system is the most important part of the entire analysis. Once such a model is available, various analytical and computer techniques can be used to analyze it. The manner in which analysis is carried out is independent of the type of physical system involved—mechanical, electrical, hydraulic, and so on.

Design. *System design* refers to the process of finding a system that accomplishes a given task. In general, the design procedure is not straightforward and will require trial and error.

Synthesis. By *synthesis*, we mean the use of an explicit procedure to find a system that will perform in a specified way. Here the desired system characteristics are postulated at the outset, and then various mathematical techniques are used to synthesize a system having those characteristics. Generally, such a procedure is completely mathematical from the start to the end of the design process.

Basic approach to system design. The basic approach to the design of any dynamic system necessarily involves trial-and-error procedures. Theoretically, a synthesis of linear systems is possible, and the engineer can systematically determine the components necessary to realize the system's objective. In practice, however, the system may be subject to many constraints or may be nonlinear; in such cases, no synthesis methods are currently applicable. Moreover, the features of the components may not be precisely known. Thus, trial-and-error techniques are almost always needed.

Design procedures. Frequently, the design of a system proceeds as follows: The engineer begins the design procedure knowing the specifications to be met and



the dynamics of the components, the latter of which involve design parameters. The specification may be given in terms of both precise numerical values and vague qualitative descriptions. (Engineering specifications normally include statements on such factors as cost, reliability, space, weight, and ease of maintenance.) It is important to note that the specifications may be changed as the design progresses, for detailed analysis may reveal that certain requirements are impossible to meet. Next, the engineer will apply any applicable synthesis techniques, as well as other methods, to build a mathematical model of the system.

Once the design problem is formulated in terms of a model, the engineer carries out a mathematical design that yields a solution to the mathematical version of the design problem. With the mathematical design completed, the engineer simulates the model on a computer to test the effects of various inputs and disturbances on the behavior of the resulting system. If the initial system configuration is not satisfactory, the system must be redesigned and the corresponding analysis completed. This process of design and analysis is repeated until a satisfactory system is found. Then a prototype physical system can be constructed.

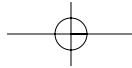
Note that the process of constructing a prototype is the reverse of mathematical modeling. The prototype is a physical system that represents the mathematical model with reasonable accuracy. Once the prototype has been built, the engineer tests it to see whether it is satisfactory. If it is, the design of the prototype is complete. If not, the prototype must be modified and retested. The process continues until a satisfactory prototype is obtained.

1-4 SUMMARY

From the point of view of analysis, a successful engineer must be able to obtain a mathematical model of a given system and predict its performance. (The validity of a prediction depends to a great extent on the validity of the mathematical model used in making the prediction.) From the design standpoint, the engineer must be able to carry out a thorough performance analysis of the system before a prototype is constructed.

The objective of this book is to enable the reader (1) to build mathematical models that closely represent behaviors of physical systems and (2) to develop system responses to various inputs so that he or she can effectively analyze and design dynamic systems.

Outline of the text. Chapter 1 has presented an introduction to system dynamics. Chapter 2 treats Laplace transforms. We begin with Laplace transformation of simple time functions and then discuss inverse Laplace transformation. Several useful theorems are derived. Chapter 3 deals with basic accounts of mechanical systems. Chapter 4 presents the transfer-function approach to modeling dynamic systems. The chapter discusses various types of mechanical systems. Chapter 5 examines the state-space approach to modeling dynamic systems. Various types of mechanical systems are considered. Chapter 6 treats electrical systems and electromechanical systems, including operational-amplifier systems. Chapter 7 deals with fluid systems,



Sec. 1-4 Summary

7

such as liquid-level systems, pneumatic systems, and hydraulic systems, as well as thermal systems. A linearization technique for nonlinear systems is explored.

Chapter 8 presents time-domain analyses of dynamic systems—specifically, transient-response analyses of dynamic systems. The chapter also presents the analytical solution of the state equation. Chapter 9 treats frequency-domain analyses of dynamic systems. Among the topics discussed are vibrations of rotating mechanical systems and vibration isolation problems. Also discussed are vibrations in multi-degrees-of-freedom systems and modes of vibrations.

Chapter 10 presents the basic theory of control systems, including transient-response analysis, stability analysis, and root-locus analysis and design. Also discussed are tuning rules for PID controllers. Chapter 11 deals with the analysis and design of control systems in the frequency domain. The chapter begins with Bode diagrams and then presents the Nyquist stability criterion, followed by detailed design procedures for lead, lag, and lag-lead compensators.

Appendix A treats systems of units, Appendix B summarizes conversion tables, and Appendix C gives a brief summary of vector-matrix algebra. Appendix D presents introductory materials for MATLAB.

Throughout the book, MATLAB is used for the solution of most computational problems. Readers who have no previous knowledge of MATLAB may read Appendix D before solving any MATLAB problems presented in this text.

