#11) Ignore wetting, ignore fiber elasticity

So Permeability $= S = \frac{r_f^2}{4k} \frac{(1-V_f)^3}{V_f^2}$

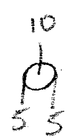
$$= \frac{(4\times10^{-6})^2}{4\times0.7} \frac{(1-0.64)^3}{(0.64)^2} = 6.51\times10^{-13}\,m^2$$

velocity (from D'arcy's law)

$$v = \frac{-S}{\eta} \frac{dP}{dz}$$

$$\eta = 8\ \text{Pa}\cdot\text{sec}$$

$$\frac{dP}{dz} \simeq \frac{F/A}{dz}$$

$F = 5\,lb = 22.4\,N$, both sides 

so $F = 44.8\,N$

$dz = 0.01\,in = 2.54\times10^{-4}\,m$

$l = \frac{1}{2}\times$ circumference $= \frac{1}{2}\times18\times\pi = 0.72\,m$

$W = \frac{1}{2}'' = 12.7\times10^{-3}\,m$

Assume the fibers are well distributed, so
$A = l_{tow}\times W_{tow}$ (sort of like a "solid")

$A = 0.72 \times 12.7 \times 10^{-3} = 9.14 \times 10^{-3}\ m$

$$\frac{dp}{dz} = \frac{\frac{44.8}{9.14 \times 10^{-3}}}{2.54 \times 10^{-4}} = 19.3 \times 10^{6}\ Pa/m$$

$$v = \frac{-S}{\eta}\frac{dp}{dz} = \frac{-6.51 \times 10^{-13}}{0.8} \times 19.3 \times 10^{6}$$

$$= 1.57 \times 10^{-5}\ m/s$$

now, it has to move 0.254 mm

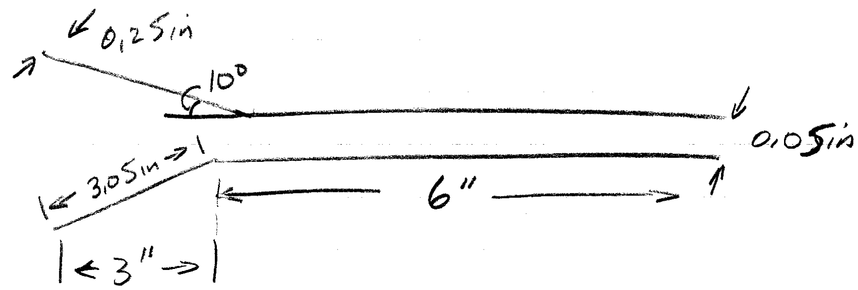
$$\text{so } t = \frac{z^2}{2}\frac{\eta}{S}\frac{1}{\Delta p} = \frac{(2.54 \times 10^{-4})^2}{2}\frac{0.8}{6.51 \times 10^{-13}} \times \frac{9.14 \times 10^{-3}}{44.8} = 8.1\ s$$

so, the fibers would need to be under pressure ~~for~~ at least 8s to wet out.
so we'd need about $\frac{1}{2}$ revolution per 8 sec
or $3\frac{3}{4}$ rpm

#12 The starting equation is (ignoring friction):

$$\frac{1}{2(w(x)+t(x))}\frac{df}{dx} = \eta\frac{V(x)}{t(x)} - (P + \sigma' \cos\phi)\sin\phi$$

The die looks like



First we need to determine where the bundle of fibers hits the taper. We do this from the initial and final volume fractions. If we assume the pultrusion fills the die, and it is 60% by volume fibers

Assuming unit length

$$\text{Die area} = 0.05 \times 0.25 = 1.25\times10^{-2}\ in^2$$
$$\text{Die area} \times v_f = 1.25\times10^{-3} \times 0.6 = 7.5\times10^{-3}\ in^2$$

so # of fibers = $\frac{\text{Area of fibers}}{\text{Area of one fiber}}$

Area of one Fiber = $\pi r^2 = \pi \times (10\times10^{-6})^2 = 3.14\times10^{-10}\,m^2$

So, the number of fibers $= \dfrac{7.5\times10^{-3}\times \overbrace{6.45\times10^{-4}}^{\text{(conversion factor)}}}{3.14\times10^{-10}}$

$= 15,398$ fibers

Now, the area at $V_f = 0.5$ (the starting point)

$$A = \frac{\text{Fiber area}}{V_f} = \frac{15398\times3.14\times10^{-10}}{0.5}$$

$$A = 9.67\times10^{-6}\,m^2$$

which gives a thickness of

$$A = w\times t$$

$$t = \frac{A}{w} = \frac{9.67\times10^{-6}}{6.35\times10^{-3}} = 1.52\times10^{-3}\,m$$

From this we can calculate the average thickness

$$\bar{t} = \frac{t_{max}+t_{min}}{2} = \frac{1.52+1.27}{2} = 1.4\,mm$$

now $$\sigma' = \frac{3\pi E}{\beta^4}\,\frac{1-\sqrt{\frac{V_f}{V_o}}}{\left(\sqrt{\frac{V_a}{V_f}}-1\right)^4}$$

$E = 10.5 \times 10^{6}\ psi = 73.4 \times 10^{9}\ Pa$

$v_a = 0.785$; $v_o = 0.5$; $v_f = 0.6$

$\beta = 220$

we'll use an average volume fraction

$$\bar{v} = \frac{v_o + v_f}{2} = \frac{0.5 + 0.6}{2} = 0.55$$

so

$$\sigma' = \frac{3 \times \pi \times 73.4 \times 10^{9}}{220^{4}} \frac{1 - \sqrt{\frac{0.55}{0.5}}}{\left(\sqrt{\frac{0.785}{0.55}} - 1\right)^{4}}$$

$$= -10{,}031\ Pa \quad \text{opposes pulling}$$

now, to find P, we need first to get the permeability

$$S = \frac{r_f^{2}}{4k} \frac{(1 - v_f)^{3}}{v_f^{2}}$$

assuming $k = 1$ and an axial flow, and $\bar{v} = 0.55$

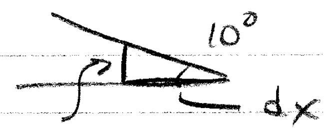
$$S = \frac{(10 \times 10^{-6})^{2}}{4 \times 1} \frac{(1 - 0.55)^{3}}{(0.55)^{2}}$$

$$= 7.53 \times 10^{-12}\ m^{2}$$

now $v = \frac{-s}{\eta} \frac{dP}{dx}$

we need v to be approximately the velocity into the die or 10 mm/s

dx is approximately the length in the axial direction of taper contact



$\frac{1.52-1.27}{2} = 0.13\,mm$

$dx = 0.74\ mm$

so $dP \sim \frac{-v\eta\,dx}{s} = \frac{10\times10^{-3} \times 0.3 \times 0.74\times10^{-3}}{7.53\times10^{-12}}$

$= -2.95\ kPa$ (opposes pulling)

so putting together in the following equation

$$df = 2\,\bar{w}\,\bar{t}\,dx\left[\eta\frac{V(x)}{\bar{t}} - (P + \sigma'\cos\phi)\sin\phi\right]$$

$$= 2\times(6.35\times10^{-3} + 1.54\times10^{-3})(0.74\times10^{-3})\left[0.3\ \frac{10\times10^{-3}}{1.4\times10^{-3}} - (-2.95\times10^{3} + -10\times10^{3}\cos 10)\sin 10\right]$$

so $df = 0.03\ N = 0.01\ lbf = 0.14\ oz$

not a very big number

Real numbers would make the value bigger, but it is so small to start, it wouldn't make much of a difference

This of course ignores the friction in the die, this would be a big number