Problem #4 Solution

To calculate the conductivity of this lay-up, we need to recognize the following relation

$$\frac{1}{k_{total}} = \frac{1}{thickness_{total}} \left[\sum_{i=1}^{n} \frac{thickness_i}{k_i} \right]$$

or

$$\frac{1}{k_{total}} = \frac{1}{t_{total}} \left[\frac{t_{steel}}{k_{steel}} + \frac{t_{prepreg}}{k_{prepreg}} + \frac{t_{\substack{bleeder \\ ply}}}{k_{\substack{bleeder \\ ply}}} + \frac{t_{\substack{vacuum \\ bag}}}{k_{\substack{vacuum \\ bag}}} \right]$$

$$t_{total} = 0.025 + 12 \times 125 \times 10^{-6} + 1.5 \times 10^{-4} + 1.3 \times 10^{-4}$$
$$= 2.678 \times 10^{-2} m$$

$k_{steel} = 45 W/mK \qquad t_{steel} = 0.025 m$

$k_{\substack{vacuum \\ bag}} = 0.293 W/mK \qquad t_{\substack{vacuum \\ bag}} = 1.3 \times 10^{-4} m$

$$k_{prepreg} = \frac{k_f k_m}{v_f k_m + v_m k_f} = \frac{1.3 \times 0.1675}{0.6 \times 0.1675 + 0.4 \times 1.3}$$

$= 0.35 \ w/mK$ $\qquad t_{prepreg} = 1.5 \times 10^{-3} m$

$k_{bleeder}$ is calculated similar to $k_{prepreg}$, assuming full wetting.

$$k_{bleeder} = \frac{k_b k_m}{v_b k_m + v_m k_b} \qquad v_b = 0.4 \quad v_m = \text{porosity} = 0.6$$

$$= \frac{1 \times 0.1675}{0.4 \times 0.1675 + 0.6 \times 1} = 0.25 \ w/mK$$

$$t_{bleeder} = 1.5 \times 10^{-4} m$$

$$\text{So } \frac{1}{k_{total}} = \frac{1}{2.678 \times 10^{-2}} \left[\frac{2.5 \times 10^{-2}}{45} + \frac{1.5 \times 10^{-3}}{0.35} + \frac{1.5 \times 10^{-4}}{0.25} + \frac{1.3 \times 10^{-4}}{0.243} \right]$$

$$= 0.22$$

$k_{total} = 4.48 \ w/mK$ (1/10 of steel)

2a) $\frac{q}{A} = u = \sigma \epsilon_{eq.} (T_1^4 - T_2^4)$

$$\frac{q_{1-2}}{A} = \sigma \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} (T_1^4 - T_2^4)$$

$$= 5.67 \times 10^{-8} \frac{1}{\frac{1}{0.95} + \frac{1}{0.85} - 1} (1273^4 - 293^4)$$

$$q_{1-2}/A = 121 \text{ kW/m}^2$$

$$h = \frac{q/A}{T_1 - T_2} = \frac{121{,}000}{1000 - 20} = 123 \text{ w/m}^2\text{-K}$$

2b) $$q_{1-2}/A = 5.67 \times 10^{-8} \frac{1}{\frac{1}{0.95} + \frac{1}{0.85} - 1} (1273^4 - 483^4)$$

$$= 119 \text{ kW/m}^2$$

$$h = \frac{\frac{q_{1-2}}{A}}{T_1 - T_2} = \frac{119{,}000}{1000 - 210} = 150 \text{ w/m}^2\text{-K}$$

So there is less energy (heat) transfered, but the heat transfer coeff is greater. The differences are pretty small, so there is little effect.

7. @ 220°C

$H_R = 218 \text{ J/g}$, $m = 20 \text{ mg}$

$\alpha = 0$ @ $t = 0$

a) time to max $\dot{H} = \frac{d\alpha}{dt} m H_R$

$\frac{d\alpha}{dt} = (K_1 + K_2 \alpha^m)(1-\alpha)^n$

$\frac{d\alpha}{dt}$ a max at $\frac{d^2\alpha}{dt^2} = 0 = (1-\alpha)^n K_2 m \alpha^{m-1} + (K_1 + K_2 \alpha^m) n (1-\alpha)^{n-1} (-1)$

$(K_1 + K_2 \alpha^m) n (1-\alpha)^{n-1} = (1-\alpha)^n K_2 m \alpha^{m-1}$

can use to calc α at $d\alpha/dt$ max

cannot use to calc. time

∴ evaluate numerically (next page)

t @ max = 4.05 min

from graph, $t \triangleq$ 7 min (due to finite heatup time)

b) max rate

$d\alpha/dt = 0.001673$ per sec

$\dot{H} = 0.001673 \text{ s}^{-1} (20 \text{ mg})(218 \text{ mJ/mg}) = 7.3 \frac{\text{mJ}}{\text{s}} = 7.3 \text{ mW}$

note $7.3/20 = 0.36$ mW/mg (matches web)

c) $\alpha = 0.325$ (see next page)

d) $\eta = 7.25$ cP (" " ")

8. (see 3rd page)

$t = 21.6$ min and $T_{peak} = 246°C$

	A	B	C	D	E	F	G	H	I	J
1	Prob 7		Comp 7793		Prob PR500	35713.00				
2										
3			alpha,0	0	intial degree of cure		A1 (s-1) =	77954		
4			alpha, f		final degree of cure		E1 (J/mol) =	76285		
5			delta t	6	time increment, s		A2 (s-1) =	1136		
6			Temperature	220	°C at start		E2 (J/mol) =	51075		
7				493	°K		m =	0.7043		
8			R =	8.314	J/mol/°K		n =	1.1607		
9			Heat Rate =	0	°C/min		Viscosity model			
10	Time, min	Alpha, %	Temp., °C	Alpha	Viscosity, P	Time, min	A1 =	-2.823		
11	0.00	0	220	0.00	0.041	0.00	A2 (1/°K) =	0.007174		
12	0.10	0	220	0.00	0.041	0.10	B1 =	2.539		
13	0.20	1	220	0.01	0.041	0.20	B2 (1/°K) =	-0.006448		
14	0.30	1	220	0.01	0.041	0.30				
15	0.40	2	220	0.02	0.042	0.40	Time, min	ln(Viscos.,P	d(alpha)/dt	
16	0.50	2	220	0.02	0.042	0.50	0.00	-3.20	0.000000	
17	0.60	3	220	0.03	0.042	0.60	0.10	-3.20	0.000644	
18	0.70	3	220	0.03	0.042	0.70	0.20	-3.19	0.000729	
19	0.80	4	220	0.04	0.043	0.80	0.30	-3.19	0.000786	
20	0.90	5	220	0.05	0.043	0.90	0.40	-3.18	0.000838	
21	1.00	5	220	0.05	0.044	1.00	0.50	-3.17	0.000885	
31	2.00	13	220	0.13	0.049	2.00	1.50	-3.08	0.001268	
41	3.00	22	220	0.22	0.058	3.00	2.50	-2.94	0.001528	
42	3.10	23	220	0.23	0.059	3.10	2.60	-2.93	0.001547	
43	3.20	24	220	0.24	0.060	3.20	2.70	-2.91	0.001564	
44	3.30	25	220	0.25	0.061	3.30	2.80	-2.89	0.001581	
45	3.40	26	220	0.26	0.062	3.40	2.90	-2.87	0.001596	
46	3.50	27	220	0.27	0.064	3.50	3.00	-2.86	0.001609	
47	3.60	28	220	0.28	0.065	3.60	3.10	-2.84	0.001621	
48	3.70	29	220	0.29	0.067	3.70	3.20	-2.82	0.001632	
49	3.80	30	220	0.30	0.068	3.80	3.30	-2.80	0.001642	
50	3.90	31	220	0.31	0.070	3.90	3.40	-2.77	0.001650	
51	**4.00**	**32**	**220**	**0.32**	**0.072**	4.00	3.50	-2.75	0.001657	
52	**4.10**	**33**	**220**	**0.33**	**0.073**	4.10	3.60	-2.73	0.001663	
53	4.20	34	220	0.34	0.075	4.20	3.70	-2.71	0.001667	
54	4.30	35	220	0.35	0.077	4.30	3.80	-2.69	0.001670	
55	4.40	36	220	0.36	0.080	4.40	3.90	-2.66	0.001672	
56	4.50	37	220	0.37	0.082	4.50	**4.00**	**-2.64**	**0.001673**	**max-**
57	4.60	38	220	0.38	0.084	4.60	**4.10**	**-2.61**	**0.001673**	**max+**
58	4.70	39	220	0.39	0.087	4.70	4.20	-2.59	0.001671	
59	4.80	40	220	0.40	0.089	4.80	4.30	-2.56	0.001668	
60	4.90	41	220	0.41	0.092	4.90	4.40	-2.53	0.001665	
61	5.00	42	220	0.42	0.095	5.00	4.50	-2.50	0.001660	
71	6.00	51	220	0.51	0.134	6.00	5.50	-2.19	0.001562	
81	7.00	60	220	0.60	0.204	7.00	6.50	-1.81	0.001401	
91	8.00	67	220	0.67	0.333	8.00	7.50	-1.35	0.001212	

Prob-8

	A	B	C	D	E	F	G	H	I	J
1			Comp 7793		Prob PR500	35713.00				
2										
3			alpha,0	0	intial degree of cure		A1 (s-1) =	77954		
4			alpha, f		final degree of cure		E1 (J/mol) =	76285		
5			delta t	6	time increment, s		A2 (s-1) =	1136		
6			Temperature	30	°C at start		E2 (J/mol) =	51075		
7				303	°K		m =	0.7043		
8			R =	8.314	J/mol/°K		n =	1.1607		
9			Heat Rate =	10	°C/min		Viscosity model			
10	Time, min	Alpha, %	Temp., °C	Alpha	Viscosity, P	Time, min	A1 =	-2.823		
11	0.00	0	30.00	0.00	0.00	0.00	A2 (1/°K) =	0.007174		
12	0.10	0	31.00	0.00	0.00	0.10	B1 =	2.539		
13	0.20	0	32.00	0.00	0.00	0.20	B2 (1/°K) =	-0.006448		
14	0.30	0	33.00	0.00	0.00	0.30				
15	0.40	0	34.00	0.00	0.00	0.40	Time, min	ln(Viscos., P	d(alpha)/dt	
16	0.50	0	35.00	0.00	0.00	0.50	0.00	-6.15	0.000000	
17	0.60	0	36.00	0.00	0.00	0.60	0.10	-6.16	0.000000	
18	0.70	0	37.00	0.00	0.00	0.70	0.20	-6.18	0.000000	
19	0.80	0	38.00	0.00	0.00	0.80	0.30	-6.20	0.000000	
20	0.90	0	39.00	0.00	0.00	0.90	0.40	-6.22	0.000000	
21	1.00	0	40.00	0.00	0.00	1.00	0.50	-6.24	0.000000	
31	2.00	0	50.00	0.00	0.00	2.00	1.50	-6.45	0.000000	
41	3.00	0	60.00	0.00	0.00	3.00	2.50	-6.73	0.000000	
51	4.00	0	70.00	0.00	0.00	4.00	3.50	-7.12	0.000000	
61	5.00	0	80.00	0.00	0.00	5.00	4.50	-7.67	0.000000	
71	6.00	0	90.00	0.00	0.00	6.00	5.50	-8.53	0.000001	
81	7.00	0	100.00	0.00	0.00	7.00	6.50	-10.07	0.000001	
91	8.00	0	110.00	0.00	0.00	8.00	7.50	-13.60	0.000002	
101	9.00	0	120.00	0.00	0.00	9.00	8.50	-29.94	0.000004	
111	10.00	0	130.00	0.00	24,066.04	10.00	9.50	26.42	0.000008	
121	11.00	0	140.00	0.00	12.91	11.00	10.50	5.02	0.000016	
131	12.00	0	150.00	0.00	1.15	12.00	11.50	1.10	0.000029	
141	13.00	1	160.00	0.01	0.35	13.00	12.50	-0.55	0.000051	
151	14.00	1	170.00	0.01	0.17	14.00	13.50	-1.45	0.000091	
161	15.00	2	180.00	0.02	0.11	15.00	14.50	-2.01	0.000158	
171	16.00	4	190.00	0.04	0.08	16.00	15.50	-2.38	0.000274	
181	17.00	7	200.00	0.07	0.06	17.00	16.50	-2.64	0.000465	
191	18.00	12	210.00	0.12	0.06	18.00	17.50	-2.81	0.000772	
201	19.00	19	220.00	0.19	0.05	19.00	18.50	-2.90	0.001230	
211	20.00	30	230.00	0.30	0.06	20.00	19.50	-2.90	0.001831	
221	21.00	45	240.00	0.45	0.07	21.00	20.50	-2.77	0.002444	
222	21.10	47	241.00	0.47	0.07	21.10	20.60	-2.75	0.002495	
223	21.20	48	242.00	0.48	0.08	21.20	20.70	-2.73	0.002543	
224	21.30	50	243.00	0.50	0.08	21.30	20.80	-2.70	0.002588	
225	21.40	52	244.00	0.52	0.08	21.40	20.90	-2.67	0.002629	
226	21.50	53	245.00	0.53	0.09	21.50	21.00	-2.64	0.002666	
227	**21.60**	**55**	**246.00**	**0.55**	**0.09**	**21.60**	21.10	-2.61	0.002698	
228	21.70	57	247.00	0.57	0.09	21.70	21.20	-2.58	0.002725	
229	21.80	58	248.00	0.58	0.10	21.80	21.30	-2.54	0.002748	
230	21.90	60	249.00	0.60	0.10	21.90	21.40	-2.50	0.002764	
231	22.00	62	250.00	0.62	0.11	22.00	21.50	-2.46	0.002775	
232	22.10	63	251.00	0.63	0.12	22.10	**21.60**	**-2.41**	**0.002781**	**max**
233	22.20	65	252.00	0.65	0.13	22.20	21.70	-2.36	0.002780	
234	22.30	67	253.00	0.67	0.14	22.30	21.80	-2.31	0.002774	
235	22.40	68	254.00	0.68	0.15	22.40	21.90	-2.25	0.002761	
236	22.50	70	255.00	0.70	0.16	22.50	22.00	-2.20	0.002742	
237	22.60	71	256.00	0.71	0.17	22.60	22.10	-2.13	0.002718	
238	22.70	73	257.00	0.73	0.19	22.70	22.20	-2.07	0.002687	
239	22.80	74	258.00	0.74	0.20	22.80	22.30	-2.00	0.002651	
240	22.90	75	259.00	0.75	0.22	22.90	22.40	-1.92	0.002609	
241	23.00	77	260.00	0.77	0.25	23.00	22.50	-1.84	0.002561	
251	24.00	88	270.00	0.88	0.81	24.00	23.50	-0.84	0.001882	
261	25.00	94	280.00	0.94	3.01	25.00	24.50	0.46	0.001117	
271	26.00	98	290.00	0.98	7.77	26.00	25.50	1.65	0.000557	