Chapter 2

P2.1 (a) $\sigma_{fu} = \left(\frac{2 E_f \gamma_f}{\pi c}\right)^{1/2}$

Since E_f and γ_f do not change, $\frac{\sigma_{fu_2}}{\sigma_{fu_1}} = \frac{c_1^{1/2}}{c_2^{1/2}}$

$$\therefore \quad c_2 = \left(\frac{\sigma_{fu_1}}{\sigma_{fu_2}}\right)^2 c_1 = \left(\frac{450{,}000}{250{,}000}\right)^2 c_1 = 3.24\ c_1$$

Longer flaw lengths in commercial fibers may be due to abrasion (rubbing against each other), handling, static fatigue etc.

(b) $c_1 = 10^{-4}$ cm $= 0.394 \times 10^{-4}$ in.

$\sigma_{fu_1} = 450{,}000$ psi, $E_f = 10.5 \times 10^6$ psi (from Table 2.1)

$$\therefore \quad \gamma_f = \frac{\pi c_1 \sigma_{fu_1}^2}{E_f} = 1.193 \ \text{lb-in/in}^2$$

P2.5

Fiber	d_f (μm)	$\epsilon_{max.}$ (%)	r_b (10^{-3} mm)
E-glass	10	4.8	104.2
S-glass	10	5	100
T-300	7	1.4	250
AS-1	8	1.32	303
AS-4	7	1.65	212
T-40	5.1	1.8	141.7
IM-7	5	1.81	138.1
HMS-4	8	0.7	571.4
GY-70	8.4	0.38	1105.3
P-55	10	0.5	1000
P-100	10	0.32	1562.5
Kevlar-49	11.9	2.8	212.5

P2.17 $\rho_{E\text{-glass}} = 2540\ kg/m^3$, $\rho_{AS\text{-}1} = 1800\ kg/m^3$ (see Table 2.1)

$\rho_{epoxy} = 1250\ kg/m^3$ (midrange value from Table 2.6)

Since $t_{E\text{-glass}} = 3\,t_{AS\text{-}1}$, $V_{\substack{E\text{-glass}\\ beam}} = 3\,V_{\substack{AS\text{-}1\\ beam}}$

Original cost with AS-1 carbon fibers,

$= (0.6)\,V_{AS\text{-}1}\,(1800\ kg/m^3)(\$40/kg) + 0.4\,V_{AS\text{-}1}\,(1250\ kg/m^3)(\$6/kg)$

$= \$\,(43{,}200 + 3{,}000)\,V_{AS\text{-}1}$

$= \$\ 46{,}200\ V_{AS\text{-}1}$

Cost with E-glass fibers

$= (0.6)(3V_{AS\text{-}1})(2540\ kg/m^3)(\$4/kg) + (0.4)(3V_{AS\text{-}1})(1250\ kg/m^3)(\$6/kg)$

$= \$\,(18{,}288 + 9{,}000)\,V_{AS\text{-}1}$

$= \$\ 27{,}288\ V_{AS\text{-}1}$

$\therefore$ % Cost Saved $= \dfrac{46{,}200 - 27{,}288}{46{,}200} \times 100 = 40.93$ ←

#5]

(a) $$l_c = \frac{d \times \sigma_{fu}}{2\tau_i}$$

$$\tau_i = \frac{\sigma_{my}}{2} = \frac{2000}{2} = 1000 \text{ psi}$$

$$d = 10 \times 10^{-6} \text{ m} = 0.000394 \text{ in}$$

So $$l_c = \frac{0.000394 \times 250{,}000}{2 \times 1000} = 0.0992 \text{ in}$$

$\simeq 125$ microns

if we use $l \simeq 50\, l_c$

we get $\underline{\underline{l \simeq 6.25 \text{ mm}}}$ $\simeq \frac{1}{4}''$

- a reasonable size

(b) $$E_{11} = v_f E_f + v_m E_m$$

$$E_{22} = \frac{E_f E_m}{v_f E_m + v_m E_f}$$

here we need v_f, v_m, but are give w_f, w_m

$$V_f = \frac{w_f/\rho_f}{\frac{w_f}{\rho_f} + \frac{(1-w_f)}{\rho_m}} = \frac{\frac{0.3}{2.5}}{\frac{0.3}{2.5} + \frac{(1-0.3)}{0.91}} = 0.135$$

so $V_m = 0.865$

hence $E_{11} = 0.135 \times 10{,}000{,}000 + 0.865 \times 200{,}000$

$= 1.523 \times 10^6$ psi

$$E_{22} = \frac{10 \times 10^6 \times 200 \times 10^3}{0.865 \times 10 \times 10^6 + 0.135 \times 200 \times 10^3}$$

$= 230{,}494$ psi

$E_{2D} = \frac{3}{8} E_{11} + \frac{5}{8} E_{22}$

$= \frac{3}{8} \times 1.523 \times 10^6 + \frac{5}{8} \times 230{,}494$

$E_{2D} = 715{,}184$ psi

(c) We know our goal is $E_{2D} = 715{,}184$ psi

First we need to calculate the new v_f, v_m

$$\text{so } v_f = \frac{\frac{0.1}{2.5}}{\frac{0.1}{2.5} + \frac{(1-0.1)}{0.93}} = 0.04 \quad , v_m = 0.96$$

so $715{,}184 = \frac{3}{8}\left[0.04 \times 10\times10^6 + 0.96 \times E_m\right]$

$$+ \frac{5}{8}\left[\frac{10\times10^6 \times E_m}{0.04 \times E_m + 0.96 \times 10\times10^6}\right]$$

solving E_m: $559{,}850$ psi

This about triples the modulus of the polypropylene, which is quite good.