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FINNED TUBE HEAT EXCHANGER OPTIMIZATION

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ABSTRACT

In this study, a system model is developed for a space-cooling system with a focus on the finned-tube condenser design details using a new environmentally friendly refrigerant as the working fluid. An optimization algorithm is implemented to find an optimum design for 10 condenser design parameters using various constraints. The figure of merit was system efficiency.

Though it is not possible to prove that the search method will give the best global design available, it did find a significantly better design than current practice. The optimum condenser design was found to give the same performance as a coil optimized through a manual search costing 23% more. It is also shown that the optimum design is consistent with minimum entropy generation for both the condenser component and the total system. This design optimization methodology is fully developed and presented in the paper so that it can be applied to other energy system's heat exchanger optimization opportunities.

INTRODUCTION

Most energy system's designs are dominated by heat exchanger performance. A primary example is the air-cooled condenser heat exchanger for single-family residential space–cooling systems with a cooling capacity that is typically approximately 10 kW. In the U.S., these residential space-cooling units must meet U.S. DOE minimum annual efficiency standards. This annual efficiency is calculated by weighting the efficiency at various outdoor temperatures with the annual operating hours at each temperature. Optimization of these heat exchangers is a complex and difficult task requiring the determination of approximately ten different finned tube heat exchanger design interrelated parameters, with appropriate constraints.

This study develops a system model for all the components of a refrigeration system. These include the evaporator, compressor, and fans, with a high fidelity model for the condenser. The condenser model incorporates available fundamental correlations for the air-side and refrigerant-side pressure drops, and heat transfer coefficients. The model calculates the refrigeration system performance under any specified air inlet temperature to the condenser and evaporator, and allows a designer to predict the refrigeration system performance for any set of condenser design parameters. These design parameters include fin thickness, fin spacing, tube diameter, tube spacing, refrigerant circuiting, coil depth, and frontal area aspect ratio. Constraints imposed are frontal area and/or condenser heat exchanger material cost.

While the effects of varying some design parameters can be studied manually using this model, an exhaustive design optimization search requires months to execute. Therefore, a design optimization search technique was implemented to optimize ten heat exchanger geometric design parameters, in addition to the air flow rate and refrigerant sub-cooling. A 9 kW cooling capacity air conditioning system was used to demonstrate the power of the methodology developed.

NOMENCLATURE

COP_{seas}	Weighted average COP over a cooling season
n	Number of parameters to be optimized
NTU	Number of transfer units
Re	Reynolds number
UA	Overall heat transfer coefficient
$\mathbf{x_i}$	One vertex or design in the Simplex
$\bar{\mathbf{x}}$	Center of the n best points
X_l	Horizontal tube spacing
X_t	Vertical tube spacing

Greek Letters

χ	Expansion	ε	Heat exchanger effectiveness
γ	Contraction	ρ	Reflection
σ	Shrinkage	τ	Multiplying factor

Subscripts

c	Contract inside	cc	Contract outside
e	Expand	r	Reflect

MODEL

Vapor Compression Cycle

The space conditioning system studied was based on a basic vapor compression refrigeration cycle, shown in Figure 1 on a Temperature-Entropy diagram. As the figure shows, low pressure, superheated refrigerant vapor from the evaporator enters the compressor (State 1) and leaves as high pressure,

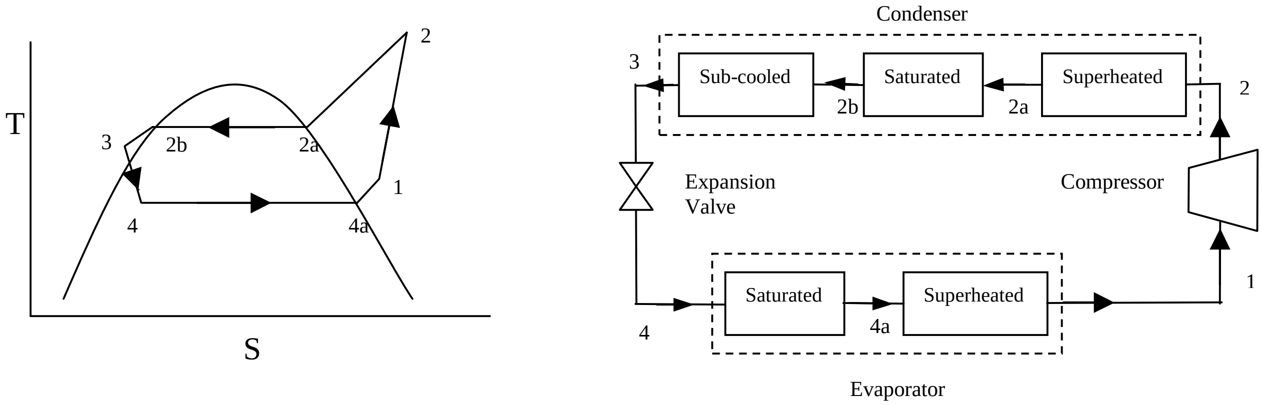

Figure 1: Vapor Compression Refrigeration Cycle

superheated vapor (State 2). This vapor enters the condenser where heat is rejected to outdoor air that is forced over the condenser coils. The refrigerant vapor is cooled to the saturation temperature (State 2b), condensed, and then cooled to below the saturation point until sub-cooled liquid is present (State 3). The high-pressure liquid then flows through the expansion valve into the evaporator (State 4) where it enters as a low pressure saturated mixture. The refrigerant is evaporated (state 4a) and then superheated by absorbing heat from warm indoor air that is blown over the evaporator coils before entering the compressor (State 1). The indoor air is cooled and dehumidified as it flows over the evaporator and returned to the living space. A complete set of the equations used to model the evaporator, compressor, valve, refrigerant mass inventory, and the basic heat exchanger equations for the condenser can be found in Wright (2000), while the condenser model will be mentioned in more detail in the following section. Entropy generation from each component was also calculated using second law relationships.

Condenser

The condenser heat exchanger used in this study is of the cross-flow, plate-fin-and-tube type. Refrigerant flows through the tubes, and a fan forces air between the fins and over the tubes. A schematic of this heat exchanger is shown in Figure 2.

When the refrigerant exits the compressor, it enters the condenser as a superheated vapor and exits as a sub-cooled liquid. In the model, the condenser is separated into three sections: superheated, saturated, and sub-cooled. In the superheated and sub-cooled sections the fluid is in a single phase, while in the saturated section two-phase flow correlations are needed. The standard heat exchanger equations used to define the relationships between heat exchanger parameters, e.g. *UA*, *NTU* and ε, can be found in Incropera & Dewitt (1996) neglecting fouling, wall thermal resistance and assuming the refrigerant side surface efficiency is one (plain tubes). The work of Zeller (1994) was used to define the fin efficiency for the hexagonal fin geometry, which provides a correlation to relate this geometry to an equivalent circular fin. Equations in Incropera & Dewitt (1996) were used to calculate the fin and surface efficiencies for this equivalent circular fin.

The most important aspect of the condenser model is the equations used to describe the heat transfer coefficients and pressure drops on both the refrigerant-side and air-side of the heat exchanger. On the airside, the heat transfer coefficient correlations used were from McQuiston (McQuiston & Parker 1994). The refrigerant side heat transfer coefficient is not as simple since there is a two-phase condition in the saturated portion of the condenser. For the single phase conditions the correlation by Kays & London (1984) was used. For the two-phase portion of the condenser, the heat transfer coefficient is calculated by the correlation of Shah (1979). Heat transfer in the tube end return bends was neglected.

To calculate the air-side pressure drop, the work of Rich (1973) was used, with the Euler number calculated from Zukauskas & Ulinskas (1998). On the refrigerant side, for the single-phase region, the pressure drop is calculated by the standard circular pipe flow pressure drop equation using a friction factor of 64/Re for laminar pipe flow and the Colebrook equation (1938) is used for turbulent pipe flow. In the saturated (two-phase) portion of the condenser, the work of

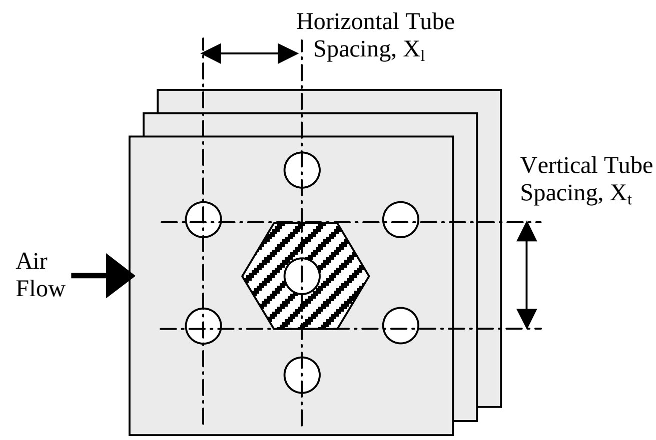

Figure 2: Finned-tube heat exchanger configuration

Hiller & Glicksman (1976) is used to find the pressure drop, which is an extension of the Lockhart & Martinelli (1948) method. The pressure drop in the tube bends is determined by the work of Chisholm (1983).

OPTIMIZATION SCHEME

Figure of Merit

The figure of merit used to quantitatively evaluate the performance of the refrigeration cycle was the Coefficient of Performance (COP). The COP is the ratio of the rate of evaporator cooling to the electrical power used to drive the system. The weighted average of the COP over a summer is referred to as the seasonal COP (COP_{seas}). Wright (2000) showed that for a space-cooling system the seasonal COP is nearly identical to the COP at an ambient temperature of 82° F (27.7˚C). Using the COP at 82˚F instead of the seasonal COP requires fewer calculations and therefore increases calculation speed and stability. In this research it is assumed that COP_{seas} is equal to the COP at 82˚F. Note that the seasonal energy efficiency rating (SEER), which is often used in the U.S. space conditioning industry, equals 3.412 times the COP_{seas}.

Simplex Search Method

In selection of an optimization algorithm the fact that the model of the air-conditioner is highly non-linear and solved numerically in EES (Engineering Equation Solver) (Klein and Alvarado 2001) must be considered. The Simplex search method presented by Nelder and Mead (1965) has been widely used to optimize complex functions. This method was chosen over more efficient techniques because it is very robust (converges consistently), relatively simple to implement, and gives good results even though it is not yet proven that the Nelder-Mead method converges to an optimum value in all cases (Lagarias, et. al., 1998). Even though the Simplex search method will find a good solution for the design of the condenser, like other search methods it cannot be proven that it is the global optimal design.

Because the Nelder-Mead algorithm uses only function values to minimize a scalar-value function of n real variables, it falls into the class of *Direct Search Methods* (Reklaits, et. al., 1983). Each kth iteration ($k \geq 0$) of the simplex direct search method begins with a simplex, specified by its n+1 vertices and the associated function values. Since the desired solution is the maximum seasonal COP of the air-conditioner, the COP_{seas} is calculated for all the vertices and they are then ordered and labeled $\mathbf{x}_1^{(k)}, \ldots, \mathbf{x}_{n+1}^{(k)}$ such that:

$$COP_{seas}(\mathbf{x}_1^{(k)}) \geq COP_{seas}(\mathbf{x}_2^{(k)}) \geq \ldots \geq COP_{seas}(\mathbf{x}_{n+1}^{(k)}) \quad [1]$$

where $\mathbf{x}_1^{(k)}$ is the *best* vertex or design in the simplex while the $\mathbf{x}_{n+1}^{(k)}$ is the worst.

To start the search, one base design is chosen. The other vertices of the starting simplex are then determine by adding τ% to one parameter at a time so the initial simplex will span the whole space. In this study the τ used was +/- 10%-30%. The percentage was decreased as the search narrowed in on the optimum point.

In the Nelder-Mead method there are four scalar parameters defined: coefficients of *reflection* (ρ), *expansion* (χ), *contraction* (γ) and *shrinkage* (σ). The recommended values by Nelder and Mead (1965), nearly universally used in the standard algorithm (Lagarias, et. al., 1998), are:

$$\rho = 1,\ \chi = 2,\ \gamma = 0.5,\ \text{and } \sigma = 0.5 \qquad [2]$$

An example of the calculations used in one iteration of the Nelder-Mead Simplex search method is shown in Table 1, while it is shown graphically in Figure 3 for the example case of a two-dimensional simplex.

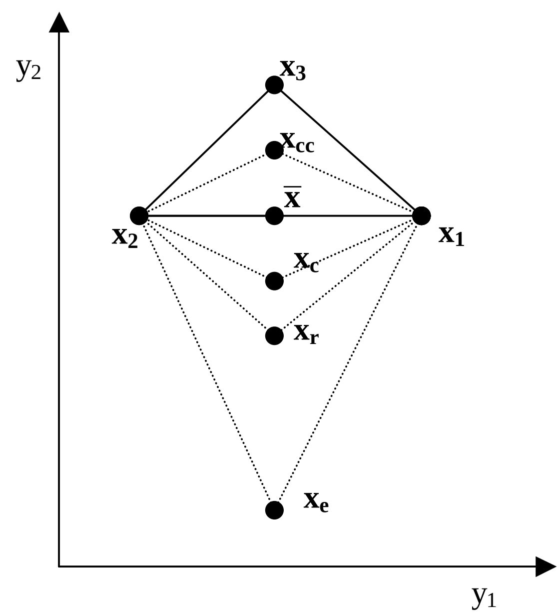

Figure 3: Nelder-Mead simplex in two dimensions with all possible new points.

Nelder and Mead did not discuss any tie-breaking rules, however, in this study points with the same value are ordered so that the newest vertex is ranked higher. The search comes to a halt when the components of the new vertex differ from the average point by less than 0.0005. This does not necessarily mean that the volume of the simplex is getting close to zero, *i.e.* the vertices are converging to the same point, but rather that the simplex is not changing between the latest two iterations. When the search comes to a stop it is restarted with the previous best point as a starting point. This is repeated until the COP_{seas} for the best point in the restarted solution is the same as the COP_{seas} for the base point.

Software Tools

The space-conditioning refrigeration cycle model used in the current study was programmed in Engineering Equation Solver, EES, (Klein and Alvarado 2001). This software tool

Table 1: Nelder-Mead Simplex search algorithm

1. Order	Order the *n+1* vertices using Equation [1] and calculate the center of the n best points: $$\bar{\mathbf{x}} = \sum_{i=1}^{n} \mathbf{x_i} / n \quad [3]$$
2. Reflect	Compute reflection point $\mathbf{x}_r$ from: $$\mathbf{x}_r = \bar{\mathbf{x}} + \rho(\bar{\mathbf{x}} - \mathbf{x}_{n+1}) = (1+\rho)\bar{\mathbf{x}} - \rho\mathbf{x}_{n+1} \quad [4]$$ Calculate COP for $\mathbf{x}_r$. -If $COP_1 \geq COP_r > COP_n$ accept $\mathbf{x}_r$ as new point and terminate the iteration.
3. Expand	If $COP_r > COP_1$ calculate the expansion point $\mathbf{x}_e$ and $COP(\mathbf{x}_e)$ where: $$\mathbf{x_e} = \bar{\mathbf{x}} + \chi(\mathbf{x}_r - \bar{\mathbf{x}}) = \mathbf{x} + \rho\chi(\bar{\mathbf{x}} - \mathbf{x}_{n+1}) = (1+\rho\chi)\bar{\mathbf{x}} - \rho\chi\mathbf{x}_{n+1} \quad [5]$$ -If $COP_e > COP_r$ accept $\mathbf{x}_e$ and terminate the iteration. -Else accept $\mathbf{x}_r$ and terminate the iteration.
4. Contract	**a.) Outside**. If $COP_n \geq COP_r > COP_{n+1}$ calculate: $$\mathbf{x}_c = \bar{\mathbf{x}} + \gamma(\mathbf{x}_r - \bar{\mathbf{x}}) = (1+\gamma\rho)\bar{\mathbf{x}} - \gamma\rho\mathbf{x}_{n+1} \quad [6]$$ -If $COP_c \geq COP_r$ then accept $\mathbf{x}_c$ and terminate the iteration. -Else perform shrinking **b.) Inside**. If $COP_r \leq COP_{n+1}$ calculate: $$\mathbf{x}_{cc} = \bar{\mathbf{x}} - \gamma(\bar{\mathbf{x}} - \mathbf{x}_{n+1}) = (1-\gamma)\bar{\mathbf{x}} + \gamma\mathbf{x}_{n+1} \quad [7]$$ -If $COP_{cc} > COP_{n+1}$ accept $\mathbf{x}_{cc}$ and terminate the iteration. -Else perform shrinking.
5. Shrink	Calculate $COP(\mathbf{v}_i)$ where: $\mathbf{v}_i = \mathbf{x}_1 + \sigma(\mathbf{x}_i - \mathbf{x}_1)$ and i = 2, …, n+1 [8] Then next simplex has vertices: $\mathbf{x}_1, \mathbf{v}_2, \ldots, \mathbf{v}_{n+1}$

iteratively solves simultaneous transcendental equations and has built-in thermodynamic and transport property relations. The model developed incorporates about 1800 simultaneous equations. While EES is useful for simulating the refrigeration cycle it is not suitable for performing the optimization Simplex search. However, EES does have to ability to communicate with other programs using Dynamic Data Exchange (DDE) supported by many other programs. Therefore, in the current study, the Simplex search scheme was programmed in Microsoft Visual Basic using Microsoft Excel to organize the inputs and the outputs of the search.

When starting the Simplex search program in Visual Basic, the *n+1* designs in the initial Simplex are transferred from the Excel sheet to the EES program. Then EES is instructed to solve the *n+1* Simplex designs. Once solved, the COP values are sent back to Excel, ordered, and the Simplex search algorithm calculates the new vertex, then this vertex is sent to EES. The COP value of the new vertex is again sent back to Excel. The Simplex search program will send vertex information from Excel to EES and the COP back to Excel until the simplex has converged. If a design parameter of a vertex doesn't fall within the constraints, the calculated seasonal COP is divided by a penalty factor. One search takes from 1 to 30 minutes to converge, depending on how close the initial design is to the optimum.

The Simplex method is not able to optimize the design using integer values for the integer parameters. However, the Simplex search method gives a solution for the optimal design of a hypothetical finned-tube condenser with decimal values that should be close to the optimal integer design. In order to find the optimal integer solution, the number of rows, number of parallel circuits and the number of tubes per circuit were manually fixed to integer values on either side of the optimal solution. The Simplex search was used to find the optimal design values for the remaining continuous parameters for each of these cases. The best case was selected as the optimum integer design.

RESULTS

COP_{seas} values are very insensitive to sub-cooling from about 5 to 20˚F (2.8 to 10˚C) with the optimum occurring around 15˚F (8.3˚C) at 95˚F (35˚C) (Wright 2000). To reduce optimization parameters and speed up calculations the sub-cool was fixed to 15˚F (8.3˚C) at 95˚F (35˚C) in the current study. In practice, the sub-cool is set by charging the system with refrigerant until a certain value of sub-cool condition is met with 95˚F (35˚C) air blowing through the coil.

Condenser coils in current space-cooling systems are dominantly produced with 5/8" (15.8mm) and 1/2" (12.7mm) diameters tubes, but 3/8" (9.5mm) and 5/16" (7.9mm) tubes are also used (AAON, 2001). The design was optimized for each of the tube diameters with constraints put on the cost and the frontal area of the condenser. The cost of the condenser coil is directly proportional to the material cost, which is calculated from the geometric design.

A fixed frontal area of 7.5 ft^2 (0.697m^2) was selected for a typical condenser design of a 9 kW space-cooling system. A maximum constraint is needed on the aspect ratio, i.e. frontal width divided by frontal height. The maximum of 3 is a common design for the condenser of a residential space-cooling system since they are often designed as a cube unit package where the condenser covers 3 sides.

Figure 4 shows that for fixed frontal area and varying condenser material cost, the smallest tube, diameter 5/16"(7.9mm), gives the best results. Smaller tubes such as 1/4"(6.35mm) are not likely to give significantly better results. Also from Figure 4 it can be seen that the optimum COP_{seas} reaches an asymptote where spending more money on a larger condenser will not result in significantly higher efficiency. As the cost constraints are increased for a condenser with a fixed frontal area, the only way to increase the efficiency is to add more rows.

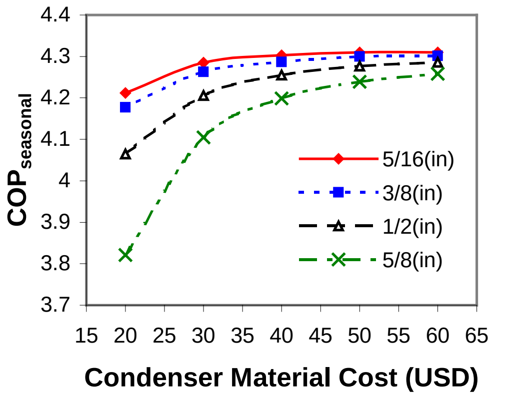

Figure 4: Seasonal COP vs. condenser material cost for varying tube diameter and fixed frontal area of 7.5 ft^2.

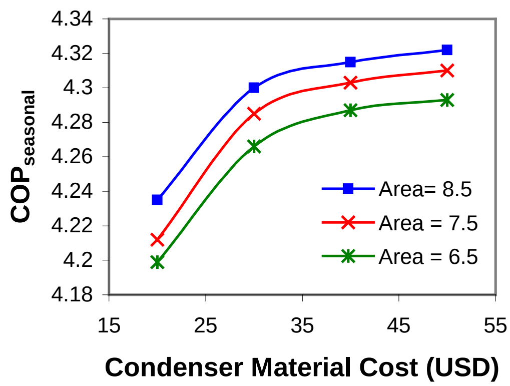

Figure 5: Seasonal COP vs. condenser material cost for varying frontal area (in ft^2) and fixed tube diameter of 5/16".

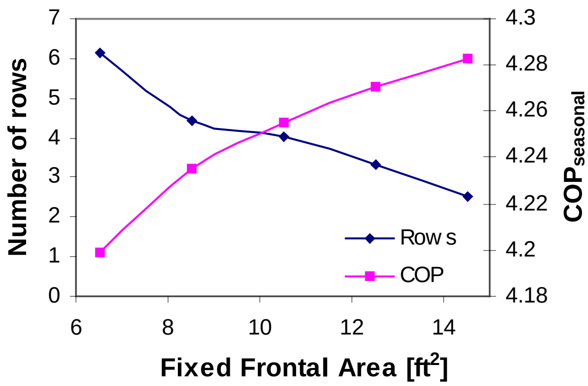

Figure 6: Number of horizontal rows and COP$_{seasonal}$ vs. frontal area with 5/16" tubing and $20 condenser material cost.

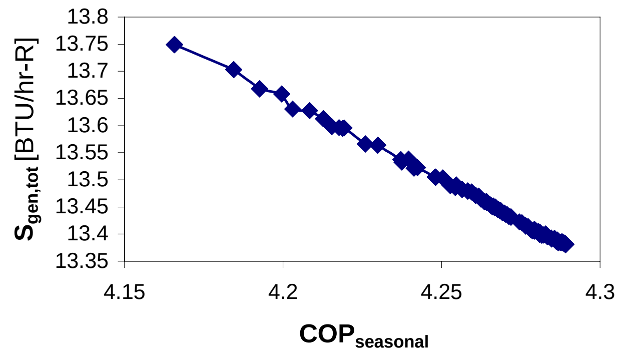

Figure 7: Total entropy generation vs. seasonal COP

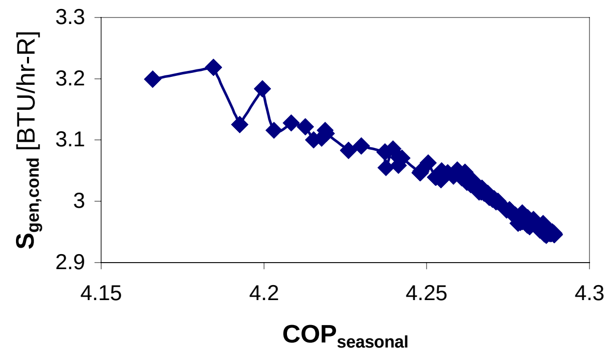

Figure 8: Condenser entropy generation vs. seasonal COP

It can be seen from Figure 5 that increasing the frontal area for a condenser with a material cost constraint makes the condenser thinner until the condenser will become just one horizontal row. This reduces the air-side pressure drop and also the air that flows across each tube would be at the ambient temperature. That would, however, make the condenser frontal area far too large for most residential applications and the size constraints are set to keep the design inside practical limits. Without any constraints the Simplex search results in the design of one straight tube, as evident from Figure 6 where the seasonal COP increases with increasing frontal area and the best design would have just one row composed of one long finned-tube. This is because there is pressure drop in the tube bends but there is no heat transfer.

Figure 7 shows a plot of the total system entropy generation vs. COP$_{seas}$. Each point designates the best design of the Simplex for each successive iteration during one design optimization. It can be concluded from this figure that there is a direct relationship between minimum entropy generation and maximum COP_{seas}. Therefore, the system could alternatively be optimized by minimizing the total system entropy generation and arrive at the same solution. Figure 8 is similar to Figure 7 except in this plot it is the condenser component entropy generation plotted vs. COP_{seas}. It can be seen from this plot that by minimizing the entropy generation in just the condenser component of the system it is likely that the same solution as maximizing COP_{seas} would also be found. The relationship is not as clear-cut as that for the total system entropy generation but it is nearly linear.

CONCLUSION

The objective of this work was to study and optimize the geometric design and operating parameters for the finned-tube condenser of a 9 kW vapor compression residential air-cooling system using R-410a as the working fluid with coil cost, aspect ratio and frontal area constraints.

It was found that the condenser with the smallest tubing, 5/16" (7.9mm) diameter, gave the best system efficiency at any cost or frontal area. Additionally, with a fixed cost constraint, larger frontal area and smaller height to width ratio results in higher efficiency until the design becomes one straight finned tube.

It was concluded that minimizing either the total system entropy generation or the condenser component entropy generation would also give a similar solution as that found with maximizing the COP_{seas}.

Using a manual search, Wright (2000) used fixed spacing between the tubes (1.25" (31.75mm) vertical and 1.08" (27.51mm) horizontal). With the Simplex search method the spacing was added as a design variable. The result is that for a fixed frontal area of 7.5 ft^2 (0.697m^2) the COP_{seas} was calculated to be 4.22 for a coil material cost of \$20. In Wright's study, the best design found for a finned-tube condenser has a COP_{seas} of 4.23 with a condenser cost of \$26. Therefore the Simplex search method found a design that gives the same COP_{seas} for 23% less cost.

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