Engineering Dynamics
Jerry H. Ginsberg
Answers to Selected Homework Problems
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1.1 $\bar{F} = 3009\bar{\imath} - 3492\bar{\jmath} + 1937\bar{k}$ N, $\bar{M}_A = 7749\bar{\imath} - 12034\bar{k}$, $M_{\text{shaft}} = -11624$ N-m.

1.4 $c_1 = -143.39$, $c_2 = -0.534501, c_3 = 440.334$ rad/s.

1.6 $F^1 = 336.8$, $F^2 = 386.2$ N.

1.8 $\bar{v}_P = [\varepsilon\alpha\cos(\alpha t)\cos\theta - R\alpha t\sin\theta]\,\bar{\imath} + [\varepsilon\alpha\cos(\beta t)\sin\theta + R\alpha t\cos\theta]\,\bar{\jmath}$,
$\theta = \frac{1}{2}\alpha t^2$, $\bar{v}_\parallel = \varepsilon\alpha\cos(\alpha t)$, $\bar{v}_\perp = R\alpha t$.

1.10
$$\bar{v} = \left[\dot{x} + L_1\dot{\theta}_1\cos\theta_1 + L_2\left(\dot{\theta}_1 + \dot{\theta}_2\right)\cos(\theta_1+\theta_2)\right]\bar{\imath} - \left[L_1\dot{\theta}_1\sin\theta_1 + L_2\left(\dot{\theta}_1 + \dot{\theta}_2\right)\sin(\theta_1+\theta_2)\right]\bar{\jmath},$$
where $\dot{x} = 1000\cos(50t)$ mm/s, $\dot{\theta}_1 = -10\pi\sin(50t)$ rad/s,
$\dot{\theta}_2 = 10\pi\cos(50t - \pi/3)$ rad/s.

2.1 $\bar{v} = [2gR\sin(s/R)]^{1/2}\,\bar{e}_t$, $\bar{a} = g\cos(s/R)\,\bar{e}_t + 2g\sin(s/R)\,\bar{e}_n$.

2.3 (a) $s = s_0/4$, (b) $s = s_0/2$, (c) $R = 2s_0/\pi$, $(d)\sin(\pi s/s_0) = (2/3)^{1/2}$.

2.6 $\bar{v} = \left(0.1451\bar{\imath} - 0.4785\bar{\jmath}\right)\beta$, $\bar{a} = \left(0.0304\bar{\imath} - 0.16973\bar{\jmath}\right)\left(\beta^2/k\right)$.

2.9
$$\bar{v} = v_0\left(1 - k\beta\sinh\eta\right)\left(\frac{\bar{\imath} + \sinh\eta\,\bar{\jmath}}{\cosh\eta}\right),$$
$$\bar{a} = -v_0^2 k\left(1 - k\beta\sinh\eta\right)\left(\frac{\bar{\imath} + \sinh\eta\,\bar{\jmath}}{\cosh\eta}\right) + \frac{v_0^2}{\beta}\left(1 - k\beta\sinh\eta\right)^2\left[\frac{-\sinh\eta\,\bar{\imath} + \bar{\jmath}}{(\cosh\eta)^3}\right].$$

2.11 $\bar{e}_t = -0.4804\bar{\imath} + 0.27740\bar{\jmath} + 0.8321\bar{k}$, $\bar{e}_n = -0.5160\bar{\imath} + 0.6769\bar{\jmath} - 0.5241\bar{k}$,
$\bar{e}_b = -0.7086\bar{\imath} - 0.6818\bar{\jmath} - 0.1818\bar{k}$, $\rho = 6.392$ m, $\tau = 40.33$ m.

2.14
$$\bar{e}_b = \frac{\rho}{(s')^3}\bar{r}' \times \bar{r}'', \quad \tau = \frac{(s')^6}{\rho^2\left(\bar{r}' \times \bar{r}''\right)\cdot\bar{r}'''},$$
$$\text{where } \rho = \frac{(s')^3}{\left[\left(\bar{r}''\cdot\bar{r}''\right)(s')^2 - \left(\bar{r}'\cdot\bar{r}''\right)^2\right]^{1/2}}.$$

2.16 $\bar{v} = L\beta\left[-\sin(\beta t)\,\bar{\imath} + \cos(\beta t)\,\bar{\jmath} - 2\sin(2\beta t)\,\bar{k}\right]$,
$\bar{a} = L\beta^2\left[-\cos(\beta t)\,\bar{\imath} - \sin(\beta t)\,\bar{\jmath} - 4\cos(2\beta t)\,\bar{k}\right]$.

2.19 (a) $x = 32.72$ m, (b) $\beta = 37.85°$. $x = 35.61$ m.

2.21 (a) $v_0 = 0.2887\left(\frac{\alpha H^3}{m}\right)^{1/2}$, (b) $t_f = 1.3095\left(\frac{m}{\alpha H}\right)^{1/2}$, $y_f = 0.8574\left(\frac{mg}{\alpha H}\right)$.

2.24 (a) $\bar{v} = u\bar{\imath} + \frac{\pi H u}{L}\cos\left(\frac{\pi x}{L}\right)\bar{\jmath}$, $\bar{a} = -\frac{\pi^2 H u^2}{L^2}\sin\left(\frac{\pi x}{L}\right)\bar{\jmath}$,
(b) $\max(v) = u\left[1 + \left(\frac{\pi H}{L}\right)^2\right]^{!/2}$ at $x = nL$, n is an integer,
(c) $\max(|\bar{a}|) = \frac{\pi^2 H u^2}{L^2}$ at $x = \frac{2n-1}{2}L$, (d) $u < \left(\frac{gL^2}{\pi^2 H}\right)^{1/2}$.

2.26 $x = \frac{\dot{y}_0}{u}\left[1 - \cos(\mu t)\right] + \frac{\dot{x}_0}{u}\sin(\mu t)$, $y = \frac{\dot{y}_0}{u}\sin(\mu t) - \frac{\dot{x}_0}{u}\left[1 - \cos(\mu t)\right]$,
$z = \dot{z}_0 t$, helical path.

2.28
$$\bar{v} = (c\beta)^{1/2}\lambda\bar{e}_R - \frac{2\pi}{3}(c\beta)^{1/2}\gamma\lambda\bar{e}_\theta + \frac{8\pi}{3}\lambda\beta\bar{e}_z,$$
$$\bar{a} = -\frac{\pi}{3}(c\beta)^{1/2}\gamma^2\lambda^2\bar{e}_R + \left(\frac{2\pi}{\sqrt{3}} - 1\right)(c\beta)^{1/2}\gamma\lambda^2\bar{e}_\theta + 2\beta\lambda^2\bar{e}_z.$$

2.31 $\bar{v} = u\cot\theta\bar{e}_R - L\Omega\sin\theta\bar{e}_\psi - u\bar{e}_z, \quad \bar{a} = -\left(\dfrac{u^2}{L(\sin\theta)^3} + L\Omega^2\sin\theta\right)\bar{e}_R - 2\Omega u\cot\theta\bar{e}_\psi.$

2.33 $N = -554.3$ N, $F_t = 999.2$ N.

2.36
$$\begin{aligned}\bar{v} =& \; ah\left(\dot{\alpha}\bar{e}_a + \dot{\beta}\bar{e}_\beta\right), \\ \bar{a} =& \; \frac{a}{h}\left[h^2\ddot{\alpha} + \left(\dot{\alpha}^2 - \dot{\beta}^2\right)(\sinh\alpha)(\cosh\alpha) - 2\dot{\alpha}\dot{\beta}(\sin\beta)(\cos\beta)\right]\bar{e}_\alpha \\ &+\frac{a}{h}\left[h^2\ddot{\beta} + \left(\dot{\alpha}^2 - \dot{\beta}^2\right)(\sin\beta)(\cos\beta) + 2\dot{\alpha}\dot{\beta}(\sinh\alpha)(\cosh\alpha)\right]\bar{e}_\beta.\end{aligned}$$

2.39 $v = 12.329$ m/s, $\dot{v} = -8.922$ m/s^2, $\rho = 3.719$ m.

2.41
$$\begin{aligned}\bar{v} =& \left(\dot{R}\cos\theta - R\dot{\theta}\sin\theta\right)\bar{\imath} + \left(\dot{R}\sin\theta + R\dot{\theta}\cos\theta\right)\bar{\jmath}, \\ a =& \left(\ddot{R}\cos\theta - 2\dot{R}\dot{\theta}\sin\theta - R\ddot{\theta}\sin\theta - R\dot{\theta}^2\cos\theta\right)\bar{\imath} \\ &+ \left(\ddot{R}\sin\theta + 2\dot{R}\dot{\theta}\cos\theta + R\ddot{\theta}\cos\theta - R\dot{\theta}^2\sin\theta\right)\bar{\jmath}.\end{aligned}$$

2.44 $v = A\omega\left[1 + 3(\sin\theta)^2\right]^{1/2}, \quad \dot{v} = 3\omega^2 A\dfrac{\sin\theta\cos\theta}{\left[1 + 3(\sin\theta)^2\right]^{1/2}}, \quad \bar{a}\cdot\bar{e}_n = \dfrac{2\omega^2 A}{\left[1 + 3(\sin\theta)^2\right]^{1/2}}.$

2.46 $\dot{r} = -350.0$ m/s, $\dot{\lambda} = 0.06239$ rad/s, $\dot{\theta} = -0.0324$ rad/s,
$\ddot{r} = 40.40$ m/s, $\ddot{\lambda} = 0.02345$ rad/s, $\ddot{\theta} = -0.00516$ rad/s.

2.49 $\bar{v} = \dot{\theta}(R'\bar{e}_R + R\bar{e}_\theta), \quad \bar{a} = \dot{\theta}^2\left[(R'' - R)\bar{e}_R + 2R'\bar{e}_\theta\right], \quad \rho = \dfrac{\left[(R')^2 + R^2\right]^{3/2}}{R''R - R^2 - 2(R')^2}.$

2.51 $v = 30.05$ m/s, $\dot{v} = -520.8$ m/s^2, $F = 40.01$.N.

2.54 $v = 20.62$ m/s, $\dot{v} = 250.0$ m/s^2

3.1
$$[R] = \begin{bmatrix} 0.8321 & -0.5547 & 0 \\ 0.3714 & 0.5571 & 0.7428 \\ -0.412 & -0.618 & 0.6695 \end{bmatrix}, \quad \bar{r}_{C/A} = 0.4828\bar{\jmath} - 0.5356\bar{k} \text{ m.}$$

3.4
$$[R] = \begin{bmatrix} -0.9285 & 0.3714 & 0 \\ -0.1564 & -0.3910 & 0.9070 \\ 0.3369 & 0.8422 & 0.4211 \end{bmatrix}, \quad \bar{r}_{O/A} = 46.42\bar{\imath} + 7.82\bar{\jmath} - 16.84\bar{k} \text{ m.}$$

3.6 (a) $[x_E \;\; y_E \;\; z_E] = [-75.09 \;\; -48.32 \;\; -42.73]$ mm,
(b) $[X_E \;\; Y_E \;\; Z_E] = [-1.62 \;\; -98.80 \;\; -6.06]$ mm.

3.9 $\beta = \cos^{-1}(0.7198\cos\theta + 0.6428\sin\theta).$

3.11 $[X_C \;\; Y_C \;\; Z_C] = [0.1465 \;\; 0.3357 \;\; 0.0766]$ m.

3.14 $\phi = 77.14°$ about $\bar{K}' = 0.9265\bar{I} - 0.3258\bar{J} - 0.1881\bar{K}$.

3.16
$$[R] = \begin{bmatrix} 0.2588 & 0.8365 & -0.4830 \\ -0.8365 & 0.4441 & 0.3209 \\ 0.4830 & 0.3209 & 0.8147 \end{bmatrix}, \quad 51.74° \text{ between original and new } y \text{ axes}$$

3.19 $\Delta\bar{r}_C = -406.9\bar{I} - 378.6\bar{J} - 505.1\bar{K}$ mm.

3.21 $\Delta\bar{r}_A = -113.19\bar{I} + 124.14\bar{J} - 13.76$ mm, $\Delta\bar{r}_B = 95.83\bar{I} - 25.66\bar{J} + 13.76\bar{K}$ mm.

3.24 $\Delta\bar{r}_C = -49.65\bar{I} - 33.59\bar{J} + 5.30\bar{K}$ mm, $\bar{v}_C(t = 0)\Delta t = -56.12\bar{I} - 23.13\bar{J} + 12.21\bar{K}$ mm.

3.26 $\bar{\omega} = 1000\pi\bar{\imath} + 0.16667\bar{k}$ rad/s, $\bar{\alpha} = 166.7\pi\bar{\jmath}$ rad/s^2,
where $\bar{\imath} = \bar{e}_t$ and $\bar{\jmath} = \bar{e}_n$ for the airplane's path.

3.29 $\theta = 90°$: $\bar{\omega} = 5236\bar{\jmath} - 20\bar{k}$ rad/s, $\bar{\alpha} = 104\,720\bar{\imath} + 100\bar{k}$ rad/s^2.
$\theta = 60°$: $\bar{\omega} = 5246\bar{\jmath} - 17.32\bar{k}$ rad/s, $\bar{\alpha} = 90\,690\bar{\imath} - 50\bar{\jmath} + 86.60\bar{k}$ rad/s^2.

3.32
$$\begin{aligned}\bar{\omega} &= -0.4330\left(\dot{\theta} + 2\dot{\beta}\right)\bar{\imath} + 0.5\left(\dot{\theta} + 2\dot{\gamma}\right)\bar{\jmath} + 0.250\left(3\dot{\theta} - 2\dot{\beta}\right)\bar{k}, \\ \bar{\alpha} &= -0.250\left(\dot{\theta}\dot{\beta} + 3\dot{\theta}\dot{\gamma} - 2\dot{\beta}\dot{\gamma}\right)\bar{\imath} - 0.8660\dot{\theta}\dot{\beta}\bar{\jmath} + 0.4330\left(\dot{\theta}\dot{\beta} - \dot{\theta}\dot{\gamma} - 2\dot{\beta}\dot{\gamma}\right)\bar{k}.\end{aligned}$$

3.34 $\bar{a}_C = \left[L\ddot{\theta}\sin 2\theta - L\dot{\theta}^2(9 + \cos 2\theta)\right]\bar{\imath} + \left[L\ddot{\theta}(3 + \cos 2\theta) + L\dot{\theta}^2\sin 2\theta\right]\bar{\jmath}, \; \bar{\imath} = \bar{e}_{C/B}.$

3.37 $\bar{a}_{B/A} = \left(-\Omega^2 H + 2\Omega\dot{\theta}W\sin\theta\right)\bar{\imath} - \left(\Omega^2 + \dot{\theta}^2\right)W(\cos\theta)\bar{\jmath} - \dot{\theta}^2 W\sin\theta\bar{k}.$

3.39 $\bar{v}_E = -(3.77L + 127.3R)\bar{\jmath} - 10(L+R)\bar{k},$
$\bar{a}_E = -(106.3L + 16315R)\bar{\imath} + 25.13(L+R)\bar{\jmath} - (495.3L + 182.1R)\bar{k}.$

3.42 $\bar{v}_D = -6.566\bar{\imath} - -2.347\bar{\jmath} + 11.638\bar{k}$ m/s, $\bar{a}_D = -314.9\bar{\imath} - 237.4\bar{\jmath} - 317.4\bar{k}$ m/s^2,
where $\bar{\jmath} = \bar{e}_{C/B}$ and $\bar{k} = \bar{e}_{B/A} \times \bar{e}_{C/B} / \left|\bar{e}_{B/A} \times \bar{e}_{C/B}\right|.$

3.45 $\bar{\omega} = \left(\frac{v}{\tau} + v\frac{d\beta}{ds}\right)\bar{e}_t + \frac{v}{\rho}\bar{e}_b,$
$\bar{\alpha} = \left(\frac{\dot{v}}{\tau} - \frac{v^2}{\tau^2}\frac{d\tau}{ds} + \dot{v}\frac{d\beta}{ds} + v^2\frac{d^2\beta}{ds^2}\right)\bar{e}_t + \frac{v^2}{\rho}\frac{d\beta}{ds}\bar{e}_n + \left(\frac{\dot{v}}{\rho} - \frac{v^2}{\rho^2}\frac{d\rho}{ds}\right)\bar{e}_b.$

3.47 $\dot{u} = (50 - \sin\theta)\, g + \dot{\theta}^2 s + \Omega^2 s (\cos\theta)^2,$
$N_{\text{horizontal}} = 2m\Omega\left(u\cos\theta - \dot{\theta}s\sin\theta\right), \quad N_{\text{vertical}} = m\left(g\cos\theta + \Omega^2\frac{s}{2}\sin 2\theta + 2\dot{\theta}u\right)$

3.50 $\bar{v}_B = \dot{\xi}\,\bar{\imath} + \Omega\xi\sin\theta\bar{\jmath} + \dot{\theta}\xi\bar{k},$
$\bar{a}_B = \left[\ddot{\xi} - \dot{\theta}^2\xi - \Omega^2\xi(\sin\theta)^2\right]\bar{\imath} + \left[2\Omega\dot{\xi}\sin\theta + 2\Omega\dot{\theta}\xi\cos\theta\right]\bar{\jmath}$
$+ \left[\ddot{\theta}\xi - \Omega^2\xi\sin\theta\cos\theta + 2\dot{\theta}\dot{\xi}\right]\bar{k}.$

3.52 $\bar{\omega} = -0.9397\Omega\cos\phi\bar{\imath} - 0.9397\Omega\sin\theta\bar{\jmath} + \left(-0.3420\Omega + \dot{\phi}\right)\bar{k},$
$\bar{\alpha} = -0.9397\Omega\dot{\phi}\sin\phi\bar{\imath} - 0.9397\Omega\dot{\phi}\cos\phi\bar{\jmath} + \ddot{\phi}\bar{k},$
$\bar{v}_G = -1.47\Omega L\sin\phi\bar{\imath} + L\left[-(0.171 + 1.47\cos\phi)\,\Omega + 0.5\dot{\phi}\right]\bar{\jmath} + 0.4698\Omega L\sin\phi\bar{k},$
$\bar{a}_G = L\left[\left(-0.5 - 0.5027\cos\phi + 0.8415(\cos\phi)^2\right)\Omega^2 - 0.5\dot{\phi}^2 + 0.342\Omega\dot{\phi}\right]\bar{\imath}$
$+L\left[0.5\ddot{\phi} + (0.5027\sin\phi - 0.2208\sin 2\phi)\,\Omega^2\right]\bar{\jmath}$
$+L\left[-(1.281 + 0.1607\cos\phi)\,\Omega^2 + 0.9397\Omega\dot{\phi}\cos\phi\right]\bar{k}$

3.55 $(\bar{v}_B)_{x_2y_2z_2} = -46.57\bar{I} + 41.16\bar{J}$ m/s, $(\bar{a}_B)_{x_2y_2z_2} = -19668\bar{I} - 35112\bar{J}$ m/s^2.

3.57 $(\bar{a}_{can})_{xyz} = (0.25s - 2.503)\, i - 0.918\bar{\jmath} - (s + 5.553)\,\bar{k}.$

3.60 $s = -\left(\frac{\omega_e\sin\lambda}{u}\right)d^2$ to the right.

3.62 (a) $x = 0, \quad y = -\omega_e\cos\lambda\left(\frac{2H}{g}\right)^{1/2}\left(\frac{2H}{3}\right),$
(b) $x = 0, \quad y = \omega_e\cos\lambda\left(\frac{2H}{g}\right)^{1/2}\left(\frac{H}{3} + R_e\right).$

4.1 $\bar{v}_D = \left(R\omega_1\cos\beta - R\omega_2 - L\dot{\beta}\right)\bar{\imath} + L\omega_1\sin\beta\bar{\jmath} - R\omega_1\sin\beta\bar{k},$
$\bar{a}_D = \left(L\omega_1^2\sin\beta\cos\beta - L\ddot{\beta}\right)\bar{\imath} + \left[-R\left(\omega_1^2 + \omega_2^2\right) + 2\omega_1\left(L\dot{\beta} + R\omega_2\right)\cos\beta\right]\bar{\jmath}$
$-\left[\omega_1^2 L(\sin\beta)^2 + L\dot{\beta}^2 + 2R\omega_2\dot{\beta}\right]\bar{k}.$

4.3 $\dot{\phi} = 97.93$ rad/s, $\dot{\theta} = -20$ rad/s, $\dot{\psi} = -48.44$ rad/s,
$\bar{\omega} = 10.12\bar{\imath} - 19.19\bar{\jmath} + 50.22\bar{k}$ rad/s.

4.6 $[R] = [R_x(\phi)]\,[R_y(\theta)]\,[R_z(\psi)],$
$\bar{\omega} = \left(\dot{\phi} - \dot{\psi}\sin\theta\right)\bar{\imath} + \left(\dot{\theta}\cos\theta + \dot{\psi}\sin\phi\cos\theta\right)\bar{\jmath} + \left(-\dot{\theta}\sin\theta + \dot{\psi}\cos\phi\cos\theta\right)\bar{k},$
$\bar{\alpha} = \left[\ddot{\phi} - \ddot{\psi}\sin\theta - \dot{\psi}\dot{\theta}\cos\theta\right]\bar{\imath} + \left[\ddot{\psi}\cos\theta\sin\phi + \ddot{\theta}\cos\phi - \dot{\psi}\dot{\theta}\sin\theta\sin\phi + \dot{\psi}\dot{\phi}\cos\theta\cos\phi\right.$
$\left.-\dot{\theta}\dot{\phi}\sin\phi\right]\bar{\jmath} + \left[\ddot{\psi}\cos\theta\cos\phi - \ddot{\theta}\sin\phi - \dot{\psi}\dot{\theta}\sin\theta\cos\phi - \dot{\psi}\dot{\phi}\cos\theta\sin\phi - \dot{\theta}\dot{\phi}\cos\phi\right]\bar{k}.$

4.8 $v = 5.313$ m/s, $\dot{v} = 724.1$ m/s^2.

4.11 $\bar{v}_A = b\dot{\theta}\cos\theta\bar{J}, \quad \bar{a}_A = b\left(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta\right)\bar{J},$
$\bar{v}_B = -b\dot{\theta}\sin\theta\bar{I}, \quad \bar{a}_B = -b\left(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta\right)\bar{I},$
$\bar{v}_G = \frac{\sqrt{2}}{2}b\dot{\theta}\cos\left(\theta + \frac{\pi}{4}\right)\left(\bar{I} + \bar{J}\right),$
$\bar{a}_G = \frac{\sqrt{2}}{2}b\left[\ddot{\theta}\cos\left(\theta + \frac{\pi}{4}\right) - \dot{\theta}^2\sin\left(\theta + \frac{\pi}{4}\right)\right]\left(\bar{I} + \bar{J}\right).$

4.13 $\theta = 60^\circ:\ \bar{\omega}_{BC} = -0.1111\dot{\theta}\bar{k},\quad \bar{\alpha}_{BC} = -0.4467\dot{\theta}^2\bar{k},\quad \bar{k}$ is outward.
$\theta = 120^\circ:\ \bar{\omega}_{BC} = -0.2727\dot{\theta}\bar{k},\quad \bar{\alpha}_{BC} = -0.1232\dot{\theta}^2\bar{k}.$

4.16 $\bar{\omega}_{BC} = 0.8660\omega_{AB}\bar{k},\ \bar{\omega}_{CD} = -0.50\omega_{AB}\bar{k},\ \bar{\alpha}_{BC} = 0.250\omega_{AB}^2\bar{k},$
$\bar{\alpha}_{CD} = 1.616\omega_{AB}^2\bar{k},\quad \bar{k}$ is outward.

4.18 $\bar{v}_B = 0.866v_A\bar{\imath} + 0.866L\Omega\bar{\jmath} + 0.5v_A\bar{k},\quad \bar{v}_G = 0.433v_A\bar{\imath} + 0.433L\Omega\bar{\jmath} - 0.25v_A\bar{k},$
where $\bar{k}$ is upward and $\bar{\imath}$ is radial.

4.21 $\bar{v}_C = -R\Omega\sin\theta\bar{\imath} + u\bar{\jmath} - u\tan\theta\bar{k},\quad \bar{a}_C = -2\Omega u\bar{\imath} - R\Omega^2\sin\theta\bar{\jmath} - \dfrac{u^2}{R}\dfrac{1}{(\cos\theta)^3}\bar{k}.$

4.23 $v_D = 18.138$ m/s, $\bar{\omega}_{CD} = 17.490\bar{\imath} + 141.22\bar{\jmath} - 90.88\bar{k}$ rad/s,
$a_D = -13802$ m/s^2, $\alpha_{CD} = 950.7\bar{\imath} + 4718\bar{\jmath} - 4940\bar{k}$ rad/s^2.

4.26 (a) No unique solution, (b) $\omega_{AB} = -\omega_{CD} = 1.20v_A\bar{J} + 1.60v_A\bar{K}$ rad/s, where $\bar{I} = \bar{e}_{A/C}$.

4.28
$$\begin{aligned}\bar{v}_P =&\ (R-r)\dot{\theta}\left[\left(1+\frac{\varepsilon}{r}\cos\phi\right)\dot{\theta}\bar{\imath} - \frac{\varepsilon}{r}\sin\phi\bar{\jmath}\right],\\ \bar{a}_P =&\ (R-r)\left[\ddot{\theta}\left(1+\frac{\varepsilon}{r}\cos\phi\right) - \frac{\varepsilon}{r}\left(\frac{R}{r}-1\right)\dot{\theta}^2\sin\phi\right]\bar{\imath}\\ &+(R-r)\left[-\ddot{\theta}\frac{\varepsilon}{r}\sin\phi + \dot{\theta}^2\left(1+\frac{\varepsilon}{r}-\frac{\varepsilon R}{r^2}\right)\cos\phi\right]\bar{\jmath}.\end{aligned}$$

4.31 $\bar{\omega}_A = 0.7273\dfrac{v}{R}$ clockwise, $\bar{\alpha}_A = 0.1172\dfrac{v^2}{R^2}$ clockwise.

4.33 $\bar{\omega} = \dfrac{v(\cos\theta)^2}{R(\cos\theta)^2 + h}$ clockwise, $\bar{\alpha} = \dfrac{2v^2h(\cos\theta)^3\sin\theta}{\left[R(\cos\theta)^2+h\right]^3}$ clockwise.

4.36 $\bar{\omega} = -\dfrac{v}{R}\bar{\imath} + \dfrac{v}{R}\cos\beta\bar{\jmath},\quad \bar{\alpha} = \dfrac{v^2}{R^2}(1+\cos\beta)\sin\beta\bar{k},$
$\bar{\imath}$ parallel to the cone generator and $\bar{\jmath}$ upward.

4.39
$$\begin{aligned}\bar{\omega} =&\ \left[\Omega_1\cos(\beta+\gamma) + (\Omega_1-\Omega_2)\frac{\sin\beta}{\sin\gamma}\right]\bar{\imath} - \Omega_1\sin(\beta+\gamma)\bar{\jmath},\\ \bar{\alpha} =&\ \left[\dot{\Omega}_1\cos(\beta+\gamma) + \left(\dot{\Omega}_1-\dot{\Omega}_2\right)\frac{\sin\beta}{\sin\gamma}\right]\bar{\imath} - (\Omega_1-\Omega_2)\Omega_1\frac{\sin\beta}{\sin\gamma}\sin(\beta+\gamma)\bar{\jmath}\\ &-\dot{\Omega}_1\sin(\beta+\gamma)\bar{k}.\end{aligned}$$

4.41 Precession : $\dot{\psi} = \Omega_1 + \dfrac{(\Omega_1-\Omega_2)}{(\sin\beta)^2}\left[\left(\dfrac{R}{r}-1\right)\cos\beta - 1\right],$
Spin: $\dot{\phi} = \dfrac{(\Omega_1-\Omega_2)}{(\sin\beta)^2}\left(\dfrac{R}{r} - 1 - \cos\beta\right),$
$\bar{\omega} = \left(\dot{\psi}\cos\beta + \dot{\phi}\right)\bar{\imath} + \dot{\psi}\sin\beta\bar{k},\quad \bar{\alpha} = -\dot{\phi}\dot{\psi}\sin\beta\bar{\jmath},$
$\bar{\imath}$ parallel to the cone generator, $\bar{k}$ upward.

4.44 $\psi = 33.69^\circ,\quad \dot{\psi} = -15.428$ rad/s, $\dot{\theta} = 0.6934$ rad/s, $\dot{\phi} = 13.699$ rad/s.

4.46
$$\begin{aligned}&\bar{\omega} = 2\Omega(1+\cos\beta)\bar{\imath} + \dot{\beta}\bar{\jmath} - \Omega\cos\beta\bar{k},\quad \dot{\beta} = \frac{u}{R}\left(\frac{1}{\sin\beta - 2\cos\beta}\right)\\ \bar{\alpha} =&\ -2\Omega\dot{\beta}\sin\beta\bar{\imath} - \left[\frac{u^2}{R^2}\frac{1}{(\sin\beta-2\cos\beta)3} + \Omega^2\cos\beta(2+2\cos\beta-\sin\beta)\right]\bar{\jmath}\\ &-2\Omega\dot{\beta}(1+\cos\beta-\sin\beta)\bar{k},\quad \bar{\imath} = \bar{e}_{C/B},\quad \bar{k}\text{ upward}.\end{aligned}$$

5.1 Initial $\bar{H}_O = 2mh\Omega\sin\theta\left(-\sin\theta\bar{I} + \cos\theta\bar{J}\right),$
$\Delta\bar{H}_O = -mh\Omega\sin2\theta\left[1.523\left(10^{-4}\right)\bar{J} + 0.01745\bar{K}\right],$
where XYZ is stationary, with X aligned with the shaft and
Z perpendicular to the initial plane of the bars.

5.6 $m = \dfrac{7}{12}\pi\rho R^2L,\quad z_G = \dfrac{11}{28}L,\quad I_{xx} = I_{yy} = m\dfrac{128L^2+93R^2}{560},\quad I_{zz} = m\dfrac{93R^2}{280}.$

5.9 $m = 6.369$ kg, $x_G = 1.1078$ m, $y_G = 0.5847$ m,
$I_{xx} = 3.544,\quad I_{yy} = 9.931,\quad I_{zz} = 13.475$ kg-m^2, $I_{xy} = 5.776$ kg-m^2.

5.11 $m = 414.1$ kg, $\bar{r}_{G/O} = 2.26\bar{\jmath} + 200\bar{k}$ mm, centroidal $\hat{x}\hat{y}\hat{z}$ are principal axes with
$I_{\hat{x}\hat{x}} = 9.495,\quad I_{\hat{y}\hat{y}} = 9.498,\quad I_{\hat{z}\hat{z}} = 9.800$ kg-m^2.

5.13 $$[I] = mR^2 \begin{bmatrix} 0.542 & 0.042 & 0 \\ 0.042 & 2.542 & 0 \\ 0 & 0 & 2.583 \end{bmatrix}.$$

5.16 $I_{xx} = 0.006667 - 0.004\left(\cos\theta\right)^2, \quad I_{yy} = 0.002667 + 0.004\left(\cos\theta\right)^2,$
$I_{xy} = 0.002\sin 2\theta$ kg-m^2.

5.19 $I_1 = 26.61\left(10^{-6}\right), \quad I_2 = 456.7\left(10^{-6}\right), \quad I_3 = 483.3\left(10^{-6}\right)$ kg-m^2.

5.21 $$[I] = \begin{bmatrix} 191.5 & 229.6 & -64.8 \\ 229.6 & 1687.5 & -11.1 \\ -64.8 & -11.1 & 1613.0 \end{bmatrix} \text{ kg-m}^2.$$

5.26 $\bar{H}_C = \frac{1}{9}mL^2\Omega\sin\theta\bar{j}, \quad d\bar{H}_C/dt = \frac{1}{9}mL^2\Omega^2\sin\theta\cos\theta\bar{k}.$

5.28 $I_{x'x'} = 0.125, \quad I_{y'y'} = 0.260, \quad I_{z'z'} = 0.225$ kg-m^2,
$\bar{H}_C = 0.1520\omega\bar{\imath} + 0.054\omega\bar{j} = 0.1118\omega\bar{\imath}' + 0.1163\omega\bar{j}'$ kg-m^2/s, $T = 0.076\omega^2$ J

5.31 $\bar{F}_A = -\frac{1}{3}mb\omega^2\bar{j}, \quad \bar{M}_A = -\frac{17}{36}mab\omega^2\bar{\imath}, \quad \bar{k} = \bar{e}_{B/A}.$

5.33 $$\bar{F}_O = \; -mL\left[\dot{\psi}^2\left(\sin\theta\right)^2 + \dot{\theta}^2\right]\bar{\imath} - mL\left[\ddot{\theta} - \dot{\psi}^2\sin\theta\cos\theta\right]\bar{j} - mL\left[\ddot{\psi}\sin\theta + 2\dot{\psi}\dot{\theta}\cos\theta\right]\bar{k},$$
$$\bar{M}_O = \; \left[I_1\ddot{\psi}\cos\theta - \left(I_1 - I_2 + I_3\right)\dot{\psi}\dot{\theta}\sin\theta\right]\bar{\imath} + \left[I_2\ddot{\psi}\sin\theta - \left(I_1 - I_2 - I_3\right)\dot{\psi}\dot{\theta}\cos\theta\right]\bar{\imath}$$
$$- \left[I_3\ddot{\theta} + \left(I_1 - I_2\right)\dot{\psi}^2\sin\theta\cos\theta\right]\bar{k}, \quad \bar{\imath} = \bar{e}_{G/O}.$$

5.36 $\bar{H}_G = mR^2\omega_1\left[0.125\lambda\bar{\imath} + \left(0.433\lambda - 0.5\right)\bar{k}\right], \quad \frac{d\bar{H}_G}{dt} = mR^2\omega_1^2\left(0.25\lambda - 0.10825\lambda^2\right)\bar{j}$
$\lambda = 2.309$ for no dynamic reactions,
$\bar{\imath}$ is the axis of the disk, $\bar{j}$ is perpendicular to the diagram.

5.39 $\bar{F}_B = 3mR\Omega^2\bar{I} + mg\bar{K}, \quad \bar{M}_B = \frac{1}{4}mR^2\left[-2\Omega\dot{\phi}\left(\cos\theta\right)^2 + \left(\Omega^2 - \dot{\phi}^2\right)\sin\theta\cos\theta\right]\bar{j}$
$\bar{\imath}$ is the axis of the disk, $\bar{j}$ is perpendicular to the diagram.

6.1 $$\Omega = \left(\frac{3g}{2L\cos\theta}\right)^{1/2}.$$

6.3 $$\bar{F}_A = \left[-mL\Omega^2\left(\frac{1}{3} + \frac{1}{9}\cos\theta\right) - \frac{1}{6}mg\right]\left(\sin\theta\right)\bar{\imath},$$
$$\bar{F}_B = \left[-mL\Omega^2\left(\frac{1}{3} - \frac{1}{9}\cos\theta\right) + \frac{1}{6}mg\right]\left(\sin\theta\right)\bar{\imath} + 2mg\bar{k},$$
$\bar{\imath}$ is radial to left, $\bar{k}$ is upward.

6.6 $$\dot{v} = 0, \quad \bar{F}_{\text{left}} = -\frac{mv^2}{6R}\cos\theta\bar{I} + \left(\frac{3}{2}mg + \frac{mv^2}{6R}\sin\theta\right)\bar{K},$$
$$\bar{F}_{\text{right}} = \frac{mv^2}{6R}\cos\theta\bar{I} + \left(\frac{3}{2}mg - \frac{mv^2}{6R}\sin\theta\right)\bar{K}, \quad \bar{K} \text{ upward.}$$

6.9 $$\bar{F}_A = \left[10g\cos\gamma - 2.5\dot{\psi}^2\left(\sin\gamma\right)^2\right]\bar{\imath} + \left[10g\sin\gamma - 2.5\dot{\psi}^2\sin\gamma\cos\gamma\right]\bar{j} - 2.5\ddot{\psi}\sin\gamma\bar{k} \text{ N},$$
$$\bar{M}_A = 0.4\ddot{\psi}\cos\gamma\bar{\imath} + 0.225\ddot{\psi}\sin\gamma\bar{j} + \left(2.5g\sin\gamma - 24\pi\dot{\psi}\sin\gamma - 0.175\dot{\psi}^2\sin\gamma\cos\gamma\right)\bar{k} \text{ N-m},$$
$\bar{\imath}$ along the axis of symmetry, $\bar{k}$ horizontal.

6.11 $\bar{F}_A = -\bar{F}_B = \frac{mR^2}{2L}\Omega_1\Omega_2\cos\theta$ parallel to the upward diameter of the disk,
$\bar{M}_1 = \frac{1}{8}mR^2\Omega_2^2\sin 2\theta\bar{e}_{A/B}.$

6.14 $$\frac{1}{3}\ddot{\theta} + \Omega^2\left(\frac{1}{2} + \frac{1}{3}\cos\theta\right)\sin\theta = \frac{g}{2L}\cos\theta.$$

6.16 $$\ddot{\xi} + \dot{\psi}^2\left[\left(\frac{L}{4} - \xi\right)\left(\sin\theta\right)^2 - \frac{L}{6}\sin\theta\cos\theta\cos\phi\right] + \frac{L}{3}\dot{\psi}\dot{\phi}\sin\theta\cos\phi = g\cos\theta,$$
$$\frac{1}{27}L\ddot{\phi} - \dot{\psi}^2\left[\frac{1}{27}L\left(\sin\theta\right)^2\sin\phi\cos\phi + \frac{1}{24}\left(L - 4\xi\right)\sin\theta\cos\theta\sin\phi\right]$$
$$-\frac{1}{3}\dot{\psi}\dot{\xi}\sin\theta\cos\phi = \frac{1}{6}g\sin\theta\sin\phi.$$

6.19 $\omega_1^2 > \dfrac{g/R}{0.4 + 1.25\sin 2\theta}$.

6.21 $N\left[L(1+\cos\gamma) - R\sin\gamma\right] = mgL(1+\cos\gamma) - mR^2\Omega^2\left[\dfrac{L^2}{R^2}\sin\gamma + \left(\dfrac{1}{4} - \dfrac{L}{2R}\right)\cos\gamma\right]\sin\gamma,\quad \gamma = \dfrac{\pi}{6}$.

6.24 $N_B = mg\cot\beta + m(R-r)\Omega_2^2 - \dfrac{2}{5}mr\dot{\psi}\dot{\phi}\cos\beta$,
$f_B = -\dfrac{2}{5}mr\dot{\phi}\dot{\psi},\quad N_A = \dfrac{mg}{\sin\beta} - \dfrac{2}{5}mr\dot{\psi}\dot{\phi}$,
where $\dot{\phi} = \dfrac{R - r - r\cos\beta}{r(\sin\beta)^2}(\Omega_1 - \Omega_2)$ and $\dot{\psi} = \Omega_2 - \dot{\phi}\cos\beta$.

6.26 Front wheel drive: $\dot{v} = \mu g\dfrac{L-b}{L+\mu h}$, rear wheel drive: $\dot{v} = \mu g\dfrac{b}{L-\mu h}$,
all wheel drive: $\dot{v} = \mu g$.

6.29 $\ddot{\theta} = 14.392\dfrac{g}{L}$.

6.31 $\ddot{\phi} = 25.02$ rad/s.

6.34 (a) $F_{\text{crit}} = \min\left(\dfrac{mg}{\sin\theta}, \dfrac{\mu mg\left(\kappa^2 + r_1^2\right)}{\kappa^2(\cos\theta + \mu\sin\theta) + \mu r_1^2\sin\theta + r_1 r_2}\right)$,
$\dot{v} = \dfrac{F}{m}\dfrac{r_1^2\cos\theta - r_1 r_2}{\kappa^2 + r_1^2}$.

6.36 $\dot{v} = \dfrac{1}{3}g,\quad \mu_{\min} = 0.2$.

6.39 $F = \sigma v$.

6.41 $\omega_A - \omega_B = \dfrac{gL}{R^2\dot{\psi}}$.

6.44 $\Delta N = 63.4$ N, increase at front wheels, decrease at rear wheels.

6.46 $v^2 = \dfrac{5.193}{1 + \kappa^2/R^2}\left(\dfrac{FR}{m}\right)$.

6.49 $\max\phi = 37.49°,\quad \dot{\phi} = 6.106$ rad/s when $\phi = 32.49°$.

6.51 $v^2 = \dfrac{2FR^3}{m(R^2 + \kappa^2)}\left[\theta + \sin\theta - \sqrt{5} + \left(8 - 2\cos\theta - (\cos\theta)^2\right)^{1/2}\right]$.

6.54 $\max\phi = 37.49°$.

6.56 (a) $\dot{\theta} = 4.3503\left(\dfrac{g}{L}\right)^{1/2}$, (b) $\max(\theta) = 133.8°$ above horizontal.

6.59 (a) $\beta = 46.56°,\quad \Omega_1 = 0.9494\left(\dfrac{g}{L}\right)^{1/2}$, .
(b) $P\Delta t = 0.1148m(gL)^{1/2},\quad v_G = 1.1123(gL)^{1/2}$.

6.61 $\omega_2 = \dfrac{mvh}{2I_A}$ where point A is the corner where impact occurs.

6.64 $\omega_2 = 4.065$ rad/s, $\bar{v}_2 = 24.70$ rad/s at $59.17°$ above left horizontal.

6.66 $(\bar{v}_G)_2 = -0.2467v\sin\theta\bar{j},\quad \omega_2 = -0.2220v\sin\theta$,
$\bar{v}_{\text{ball}} = (-\cos\theta\bar{\imath} - 0.01328\sin\theta\bar{j})v$.

6.69 $(\bar{v}_B)_2 = 0.8613v_1$ at $50.74°$ below the left direction.

6.71 $\bar{v}_G = 15.459$ m/s downward, $\omega_2 = 17.441$ rad/s counterclockwise.

6.74 $\bar{v}_2 = 16.57\bar{\imath} + 1.33\bar{j} + 6\bar{k}$ m/s, $\bar{\omega}_2 = 33.7\bar{\imath} + 484.3\bar{j} + 50\bar{k}$ rad/s.

7.1 $2\beta(X - ut)\dot{X} - \dot{Y} - 2\beta(X - ut)u = 0,\quad Y = \beta(X - ut)^2$.

7.3 $R\dot{\theta} + \dot{s} - \dot{X}_C\cos\theta + \dot{Y}_C\sin\theta = 0,\quad s\dot{\theta} + \dot{X}_C\sin\theta + \dot{Y}_C\cos\theta = 0$,
both constraints are holonomic.

7.6 $\dot{x}\cos\theta - \dot{\theta}x\sin\theta = 0$.

7.8 $\dot{\theta}_1\sin\theta_1 + \dot{\theta}_2\sin\theta_2 + \dot{\theta}_3\sin\theta_3 = 0,\quad \dot{\theta}_1\cos\theta_1 - \dot{\theta}_2\cos\theta_2 - \dot{\theta}_3\cos\theta_3 = 0$.

7.11 $\dot{x}_A\sin(\theta + \beta) - L\dot{\theta}\cos\beta = 0$.

7.14 $(\sin\beta - 2\cos\beta) R\dot{\beta} - u = 0, \quad [2(1+\cos\beta) - \sin\beta]\dot{\psi} + \dot{\phi} = 0.$

7.16 $q_1 = y: \quad Q_1^{\text{cons}} = -k(y - 1.5L) - 8\sigma L, \quad Q_1^{\text{nc}} = -\dfrac{F\left(3y^2 + 4L^2\right) + 2ML}{2L\left(4L^2 - y^2\right)^{1/2}},$

$q_1 = \theta: \quad Q_1^{\text{cons}} = -2kL^2(2\sin\theta - 1.5)\cos\theta - 8\sigma L^2\cos\theta,$

$Q_1^{\text{nc}} = -FL\left[2 + 6(\sin\theta)^2\right] - M.$

7.19 $\delta W = -N_B \sin(\theta+\beta)\,\delta x_A + N_B L\delta\theta\cos\beta,$

$\delta x_A = L\delta\theta\dfrac{\cos\beta}{\sin(\theta+\beta)}$ if kinematically admissible.

7.21 $V = -\left(x^2 - y^2\right)^{1/2}.$

7.24 $$m\left[\frac{1}{3}L^2 + \frac{H^2}{(\cos\theta)^4}\right]\ddot{\theta} + 2m\frac{H^2\sin\theta}{(\cos\theta)^5}\dot{\theta}^2 + \frac{1}{2}mgL\cos\theta + \frac{mgH}{(\cos\theta)^2} + kH^2\frac{\left(\tan\theta - \tan\frac{\pi}{9}\right)}{(\cos\theta)^2} = -\Gamma,$$

$\theta = 16.092^\circ$ for static equilibrium.

7.26 $3m\ddot{X} + m\ddot{s}\cos\theta + 2kX = 0, \quad m\ddot{s} + m\ddot{X}\cos\theta + ks - mg\sin\theta = 0.$

7.29 $m\left[2\ddot{R}_1 + \ddot{R}_2 - (2R_1 + R_2)\dot{\theta}^2\right] + k(R_1 - L) - 2mg\cos\theta = 0,$

$m\left[\ddot{R}_1 + \ddot{R}_2 - (R_1 + R_2)\dot{\theta}^2\right] + k(R_2 - L) - mg\cos\theta = 0.$

7.31 $\dfrac{8}{3}mL^2\ddot{\theta} + kL^2\left(4\sin\theta - \dfrac{3}{2}L\right)\cos\theta = 2(F + mg)L\cos\theta.$

7.34 $\dfrac{1}{3}mL^2\ddot{\theta} + \dfrac{1}{2}mL(\ddot{u} + g)\sin\theta = 0.$

7.36 $\dfrac{1}{3}mL^2\ddot{\phi} + \dfrac{1}{2}mL\varepsilon\Omega^2\sin\phi + k\phi = 0.$

7.39 $4mR^2\left(\ddot{\theta} + \omega^2\sin\theta\cos\theta\right) + 2mgR\sin(2\theta + \omega t) = 2FR\sin\theta.$

7.41 $\left[1 + 8(\cos\theta)^2\right]\ddot{\theta} - \left(9\Omega^2 + 8\dot{\theta}^2\right)\sin\theta\cos\theta = \dfrac{(2mg - 4F)}{mL}\sin\theta.$

7.44 $(m_1 + m_2)\ddot{x} - m_2(R - r)\left(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta\right) + kx = F,$

$\dfrac{3}{2}(R - r)\ddot{\theta} - \ddot{x}\cos\theta + g\sin\theta = 0.$

7.46 $\left(\dfrac{1}{12}b^2 + \dfrac{1}{3}h^2 + R^2\theta^2\right)\ddot{\theta} + R^2\theta\dot{\theta}^2 + g\left(R\theta\cos\theta - \dfrac{h}{2}\sin\theta\right) = 0.$

7.48 $(m_1 + m_2)\ddot{s} + m_1(R + \varepsilon\cos\theta)\ddot{\theta} - m_1\varepsilon\dot{\theta}^2\sin\theta = (m_1 + m_2)g\sin\beta,$

$$m_1\left(R^2 + \varepsilon^2 + \kappa^2 + 2R\varepsilon\cos\theta\right)\ddot{\theta} + m_1(R + \varepsilon\cos\theta)\ddot{s} - m_1R\varepsilon\dot{\theta}^2\sin\theta = m_1 g\left[R\sin\beta + \varepsilon\sin(\beta + \theta)\right].$$

7.51 $I\ddot{\theta} + \dfrac{1}{2}\dfrac{dI}{d\theta}\dot{\theta}^2 + \dfrac{3}{2}mgL\cos\theta = FL\dfrac{\cos(\beta - \theta)}{\sin\beta}$

$$\text{where } I = mL^2\left\{\frac{1}{12} + \frac{1}{(\sin\beta)^2}\left[\left(\frac{5}{4} + \frac{\kappa^2}{R^2}\right)\left((\cos\theta)^2 + \cos(\beta - \theta)^2\right) + \frac{1}{2}\cos\theta\cos(\beta - \theta)\cos\beta\right]\right\}$$

7.54 $m_1\ddot{s} - m_1\left[\dot{\theta}^2 + \dot{\psi}^2(\sin\theta)^2\right]s + ks + m_1 g\sin\theta\cos\psi = 0,$

$\left(I_1 + m_1 s^2\right)\ddot{\theta} + 2m_1 s\dot{s}\dot{\theta} - m_1 s^2\dot{\psi}^2\sin\theta\cos\theta + m_1 gs\cos\theta\cos\psi = 0,$

$\left[I_2 + m_1 s^2(\sin\theta)^2\right]\ddot{\psi} + 2m_1\dot{\psi}s\left[\dot{s}(\sin\theta)^2 + s\dot{\theta}\sin\theta\cos\theta\right] - m_1 gs\sin\theta\sin\psi = \Gamma.$

7.56 $\dfrac{1}{4}mR^2\left[\left(1 + (\cos\theta)^2\right)\dot{\Omega}_1 + 2\dot{\Omega}_2\cos\theta\right] = C.$

7.58 $$\frac{2}{3}mL^2\ddot{\theta}+\frac{1}{\sqrt{2}}mgL\sin\theta=0,\quad \frac{1}{3}mL^2\ddot{\psi}=M.$$

7.61 $$\ddot{\xi}-\Omega^2\xi\left(\sin\theta\right)^2-\left(\xi-\frac{L}{2}\right)\dot{\theta}^2-g\cos\theta=0,$$
$$\left(\frac{1}{3}L^2-L\xi+\xi^2\right)\ddot{\theta}+\left(2\xi-L\right)\dot{\xi}\dot{\theta}-\Omega^2\left(\xi^2+\frac{1}{12}L^2\right)\sin\theta\cos\theta+g\left(\xi-\frac{L}{2}\right)\sin\theta=0.$$

7.63 $$\left(m_1+m_2\right)\ddot{z}+\frac{1}{2}m_2L\left(\ddot{\theta}\sin\theta+\dot{\theta}^2\cos\theta\right)+\left(m_1+m_2\right)g+kz=F,$$
$$\frac{1}{3}L\ddot{\theta}+\frac{1}{2}\ddot{z}\sin\theta-\frac{1}{3}L\dot{\psi}^2\sin\theta\cos\theta+\frac{1}{2}g\sin\theta=0,$$
$$\frac{1}{3}m_2L^2\left[\ddot{\psi}\left(\sin\theta\right)^2+2\dot{\psi}\dot{\theta}\sin\theta\cos\theta\right]=M.$$

7.66 $$\left[I_p+m\left(L^2+\frac{1}{4}R^2\right)\left(1+\left(\cos\beta\right)^2\right)+2mL^2\cos\beta\right]\ddot{\psi}$$
$$-2m\sin\beta\left[L^2+\left(L^2+\frac{1}{4}R^2\right)\cos\beta\right]\dot{\beta}\dot{\psi}+\frac{1}{2}mR^2\dot{\phi}\dot{\beta}\sin\beta=\Gamma,$$
$$m\left(L^2+\frac{1}{4}R^2\right)\ddot{\beta}+m\sin\beta\left[L^2+\left(L^2+\frac{1}{4}R^2\right)\cos\beta\right]\dot{\psi}^2-\frac{1}{2}mR^2\dot{\psi}\dot{\phi}\sin\beta-mgL\cos\beta=0.$$

8.2 $$m\left[R^2+\kappa^2+2R^2\left(1+\sin\theta\right)^2\right]\ddot{\theta}+3.75mR^2\left(1+\sin\theta\right)\left[\ddot{\phi}\sin\phi+\dot{\phi}^2\cos\phi\right]$$
$$+2mR^2\dot{\theta}^2\left(1+\sin\theta\right)\cos\theta=FR\left(1+\sin\theta\right)+\lambda_1\cos\theta,$$
$$\frac{25}{12}mR^2\left\{\left[1+3\left(\sin\phi\right)^2\right]\ddot{\phi}+3\dot{\phi}^2\sin\phi\cos\phi\right\}+3.75mR^2\sin\phi\left[\ddot{\theta}\left(1+\sin\theta\right)+\dot{\theta}^2\cos\theta\right]$$
$$+1.25mgR\cos\phi=2.5FR\sin\phi-2.5\lambda_1\cos\phi,$$
$$\dot{\theta}\cos\theta-2.5\dot{\phi}\cos\phi=0$$

8.4 $$\{z\}=\begin{bmatrix}\psi & \beta & \dot{\psi} & \dot{\beta}\end{bmatrix}^{\mathrm{T}},\quad I_1=m\left[L^2+\kappa_1^2\left(\sin\beta\right)^2+\kappa_2^2\left(\cos\beta\right)^2\right]$$
$$\begin{bmatrix} I_1 & 0 & -c_1\\ 0 & m\kappa_2^2 & 1\\ -c_1 & 1 & 0\end{bmatrix}\begin{Bmatrix}\ddot{\psi}\\ \ddot{\beta}\\ \lambda_1\end{Bmatrix}=\begin{Bmatrix}-m\left(\kappa_1^2-\kappa_2^2\right)\dot{\psi}\dot{\beta}\sin2\beta+m\kappa_1^2\Omega_1\dot{\beta}\cos\beta+\Gamma\\ \frac{1}{2}m\left(\kappa_1^2-\kappa_2^2\right)\dot{\psi}^2\sin2\beta+m\kappa_1^2\Omega_1\dot{\psi}\cos\beta\\ c_2\dot{\psi}\end{Bmatrix}$$

8.6 $$\left(m_1+m_2\right)\ddot{x}-m_2\left(\ddot{R}-R\dot{\theta}^2\right)\cos\theta+m_2\left(R\ddot{\theta}+2\dot{R}\dot{\theta}\right)\sin\theta+2k_1x=\lambda_1\sin\theta,$$
$$m_2\left(\ddot{R}-\ddot{x}\cos\theta-R\dot{\theta}^2\right)+k_2\left(R-R_0\right)=0$$
$$m_2\left(R\ddot{\theta}+\ddot{x}\sin\theta+2\dot{R}\dot{\theta}\right)-m_2g\cos\theta=\lambda_1,\quad \dot{x}\sin\theta+R\dot{\theta}=0.$$

8.9 $$m_1R^2\left\{\left[4.5+8\cos\theta+3.75\left(\cos\theta\right)^2\right]\ddot{\psi}+\frac{1}{2}\left(\sin\theta\right)\ddot{\phi}-\left(8+7.5\cos\theta\right)\left(\sin\theta\right)\dot{\psi}\dot{\theta}\right.$$
$$\left.+\frac{1}{2}\dot{\phi}\dot{\theta}\cos\theta\right\}=M+\lambda_1R\left(\sin\theta-2-2\cos\theta\right),$$
$$\left\{4.25m_1R^2+m_2R^2\left[1-2\sin2\theta+3\left(\cos\theta\right)^2\right]\right\}\ddot{\theta}-\frac{1}{2}m_2R^2\left(4\cos2\theta+3\sin2\theta\right)\dot{\theta}^2$$
$$+m_1R^2\left(4+3.75\cos\theta\right)\left(\sin\theta\right)\dot{\psi}^2-2\left(m_1+m_2\right)gR\cos\theta+m_2gR\sin\theta$$
$$=FR\left(\sin\theta-2\cos\theta\right),$$
$$\frac{1}{2}m_1R^2\left(\ddot{\phi}+\ddot{\psi}\sin\theta+\dot{\psi}\dot{\theta}\cos\theta\right)=\lambda_1R,\quad R\left(\sin\theta-2-2\cos\theta\right)\dot{\psi}+R\dot{\phi}=0.$$

8.11 $$\left(m_A+m_{AB}+m_B\right)\ddot{x}_A-\frac{1}{2}\left(m_{AB}+3m_B\right)L\left(\ddot{\theta}\sin\theta+\dot{\theta}^2\cos\theta\right)=F+\lambda_1\sin\left(\beta+\theta\right),$$
$$\left(\frac{1}{3}m_{AB}L^2+\frac{3}{2}m_BL^2+\frac{1}{4}m_BR^2\right)\ddot{\theta}-\frac{1}{2}\left(m_{AB}+3m_B\right)L\ddot{x}_A\sin\theta$$
$$=-\lambda_1L\cos\beta,\quad \dot{x}_A\sin\left(\beta+\theta\right)+\dot{\theta}L\cos\beta=0.$$

8.13 $q_1 = X_C,\quad q_2 = Y_C,\quad q_3 = \theta,\quad \dot{X}_C \sin\theta - \dot{Y}_C \cos\theta = 0$

$$\left(m + \frac{3}{2}m_w\right)\ddot{X}_C + mL\left(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta\right) = \left(\bar{F}_1 + \bar{F}_2\right)\cdot\bar{I} + \lambda_1\sin\theta$$

$$\left(m + \frac{3}{2}m_w\right)\ddot{Y}_C - mL\left(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta\right) = \left(\bar{F}_1 + \bar{F}_2\right)\cdot\bar{J} - \lambda_1\cos\theta$$

$$\left(I + m_1L^2 + \frac{1}{4}m_wR^2\right)\ddot{\theta} + mL\left(\ddot{X}_C\sin\theta - \ddot{Y}_C\cos\theta\right) = \bar{r}_{A/C}\times\bar{F}_1 + \bar{r}_{B/C}\times\bar{F}_2$$

8.15 $4m\ddot{s} + 2mL\ddot{\theta}\sin\theta + 2mL\dot{\theta}^2\cos\theta = F + \lambda_2\sin\Omega t,$

$$\left[\frac{1}{6} + \frac{3}{2}(\sin\psi)^2(\sin\theta)^2\right]\ddot{\psi} - \frac{3}{8}\ddot{\theta}\sin 2\psi\sin 2\theta + \frac{3}{2}\dot{\psi}^2\sin\psi\cos\psi(\sin\theta)^2 + \frac{3}{4}\dot{\theta}^2\sin 2\psi$$
$$+\frac{3}{2}\dot{\psi}\dot{\theta}(\sin\psi)^2\sin 2\theta = \frac{1}{mL}\left(\lambda_1\cos\psi\sin\theta - \lambda_2\cos\Omega t\sin\psi\sin\theta\right),$$

$$mL^2\left[\frac{1}{6} + \frac{3}{2}(\sin\theta)^2 + \frac{3}{2}(\cos\psi)^2(\cos\theta)^2\right]\ddot{\theta} + 2mL\ddot{s}\sin\theta$$
$$+\frac{3}{2}mL^2\dot{\theta}^2\sin 2\theta\left[1 + (\cos\psi)^2\right] - \frac{1}{2}mL^2\dot{\psi}^2\left[\frac{1}{3} + 3(\sin\psi)^2\right]\sin 2\theta$$
$$= \lambda_1L\sin\psi\cos\theta + \lambda_2L\left[\sin\Omega t\sin\theta + \cos\Omega t\cos\psi\cos\theta\right],$$

$$L\dot{\psi}\cos\psi\sin\theta + L\dot{\theta}\sin\psi\cos\theta = 0,$$

$$\dot{s}\sin\Omega t - L\dot{\psi}\cos\Omega t\sin\psi\sin\theta + L\dot{\theta}\left[\sin\Omega t\sin\theta + \cos\Omega t\cos\psi\cos\theta\right]$$
$$+\Omega\left[s\cos\Omega t - L\sin\Omega t\cos\psi\sin\theta - L(\cos\Omega t)^2\right] = 0.$$

8.17 $X_G = -111.2$ m, $Y_G = -70.0$ m, $\theta = 56.2°$ at $t = 60$ s.

8.19 θ passes 89° when $t = 9.28$ s.

8.22 $\max(\theta) = 80.73°$ when $t = 1.995$ s.

8.25 $\frac{1}{2}\ddot{\theta} + \frac{4}{3\pi}\frac{g}{R}\sin\theta + \mu\left[\frac{g}{R} - \frac{4}{3\pi}\left(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta\right)\right]\mathrm{sgn}\left(\dot{\theta}\right) = 0.$

8.28 $\frac{1}{3}mL^2\ddot{\phi} + \frac{1}{2}ml\ddot{Y}_B\sin\phi - \frac{1}{2}mgL\sin\phi = \mu\left|N_A\right|(\cos\phi)\,\mathrm{sgn}\left(\dot{\phi}\right) - N_A\sin\phi,$

$$m\ddot{Y}_B(\sin\phi)^2 + \frac{1}{2}mL\ddot{\phi}\sin\phi + \frac{1}{2}mL\dot{\phi}^2\cos\phi = -\frac{H}{\sin\phi}N_A,\quad \dot{Y}_B(\sin\phi)^2 + H\dot{\phi} = 0.$$

8.31 $\{q\} = [\phi\;\; X_B\;\; Y_B]^{\mathrm{T}},\quad \{x\} = \left[\{q\}^{\mathrm{T}}\;\;\{\dot{q}\}^{\mathrm{T}}\right]^{\mathrm{T}},\quad \frac{d}{dt}\{x\} = \left[\{\dot{q}\}^{\mathrm{T}}\;\;\{\ddot{q}\}^{\mathrm{T}}\right]^{\mathrm{T}},$

$$\begin{bmatrix}[M] & -[B]\\ -[a] & [0]\end{bmatrix}\left\{\begin{array}{c}\{\ddot{q}\}\\ \left\{\begin{array}{c}N_A\\ N_B\end{array}\right\}\end{array}\right\} = \left\{\begin{array}{c}\{F\}\\ [\dot{a}]\{a\}\{\dot{q}\}\end{array}\right\},$$

$$[M] = \begin{bmatrix}L^2/3 & (L/2)\cos\phi & (L/2)\sin\phi\\ (L/2)\cos\phi & 1 & 0\\ (L/2)\sin\phi & 0 & 1\end{bmatrix},$$

$$[B] = \begin{bmatrix}-H/\sin\phi & 0\\ -\left(\mu\sin\phi\,\mathrm{sgn}\left(\dot{\phi}\right) + \cos\phi\right) & 1\\ \left(\mu\cos\phi\,\mathrm{sgn}\left(\dot{\phi}\right) - \sin\phi\right) & 0\end{bmatrix},\quad \{F\} = \left\{\begin{array}{c}(L/2)\,g\sin\phi\\ (L/2)\,\dot{\phi}^2\sin\phi\\ g - (L/2)\,\dot{\phi}^2\cos\phi\end{array}\right\},$$

$$[a] = \begin{bmatrix}0 & 1 & 0\\ H & 0 & (\sin\phi)^2\end{bmatrix},\quad [\dot{a}] = \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & \dot{\phi}\sin 2\phi\end{bmatrix},$$

$N_B = 0$ at $t = 0.237$ s, $\phi = 45.54°$.

9.2 $\mu\ddot{w} + (EIw'')'' + \mu g - f = 0$; either w is specified or $(EIw'')' = 0$ at $x = 0$ and $x = L$; either w' is specified or $EIw'' = 0$ at $x = 0$ and $x = L$.

9.5 $-\mu\ddot{w}_y + F\left\{\left[1 - \frac{3}{2}\left(\frac{\partial w_y}{\partial x}\right)^2 - \frac{1}{2}\left(\frac{\partial w_z}{\partial x}\right)^2\right]\frac{\partial^2 w_y}{\partial x^2} - \frac{\partial w_y}{\partial x}\frac{\partial w_z}{\partial x}\frac{\partial^2 w_z}{\partial x^2}\right\} - \mu g + f_y = 0,$

$$-\mu\ddot{w}_z + F\left\{\left[1 - \frac{3}{2}\left(\frac{\partial w_z}{\partial x}\right)^2\frac{\partial^2 w_z}{\partial x^2} - \frac{1}{2}\left(\frac{\partial w_y}{\partial x}\right)^2\right]\frac{\partial^2 w_z}{\partial x^2} - \frac{\partial w_y}{\partial x}\frac{\partial w_z}{\partial x}\frac{\partial^2 w_y}{\partial x^2}\right\} + f_z = 0.$$

9.7 $$\frac{1}{2}\mu L\ddot{q}_1 + \frac{\pi^2 F}{2L}q_1 - \frac{\pi^4 F}{16L^3}\left(3q_1^3 + 24q_1q_2^2\right) + \frac{2}{\pi}\mu g L = 0,$$
$$\frac{1}{2}\mu L\ddot{q}_2 + \frac{4\pi^2 F}{L}q_2 - \frac{3\pi^4 F}{2L^3}\left(q_1^2q_2 + 2q_2^3\right) = 0.$$

9.10 $$\frac{1}{2}\mu L\ddot{p}_1 + \frac{\pi^2 F}{2L}p_1 - \frac{\pi^4 F}{L^3}\left[\frac{3}{16}p_1^3 + \frac{3}{16}p_1q_1^2 + \frac{3}{2}p_1p_2^2 + \frac{1}{2}p_1q_2^2 + q_1p_2q_2\right]$$
$$+\frac{2}{\pi}\mu g L = \frac{2}{\pi}f_0 L\sin\left(\Omega t\right),$$
$$\frac{1}{2}\mu L\ddot{q}_1 + \frac{\pi^2 F}{2L}q_1 - \frac{\pi^4 F}{L^3}\left[\frac{3}{16}q_1^3 + \frac{3}{16}p_1^2q_1 + \frac{1}{2}q_1p_2^2 + \frac{3}{2}q_1q_2^2 + p_1p_2q_1\right] = 0,$$
$$\frac{1}{2}\mu L\ddot{p}_2 + \frac{2\pi^2 F}{L}p_2 - \frac{\pi^4 F}{L^3}\left[\frac{3}{2}p_1^2p_2 + \frac{1}{2}q_1^2p_2 + p_1q_1q_2 + 2p_2^3 + 2p_2q_2^2\right] = 0,$$
$$\frac{1}{2}\mu L\ddot{q}_2 + \frac{2\pi^2 F}{L}q_2 - \frac{\pi^4 F}{L^3}\left[\frac{1}{2}p_1^2q_2 + \frac{3}{2}q_1^2q_2 + p_1q_1p_2 + 2q_2^3 + 2p_2^2q_1\right] = 0.$$

9.12 $$m\dot{x}_1 = \frac{p_1 - p_2\cos\theta}{5 - (\cos\theta)^2}, \quad m\dot{x}_2 = \frac{-p_1\cos\theta + 5p_2}{5 - (\cos\theta)^2}, \quad \dot{p}_1 = 0, \quad \dot{p}_2 = m_2 g\sin\theta - kx_2.$$

9.15 $$\dot{\psi} = \frac{p_1}{\Delta}, \quad \dot{\beta} = \frac{p_2}{I_2}, \quad \dot{p}_1 = -mg\left(\frac{H}{2} - w\right)\sin\beta\cos\psi,$$
$$\dot{p}_2 = -\frac{p_1^2}{2\Delta^2}\left(I_1 - I_2\right)\sin 2\beta - mg\left(\frac{H}{2} - w\right)\cos\beta\sin\psi,$$
$$\Delta = I_1\left(\cos\beta\right)^2 + I_2\left(\sin\beta\right)^2.$$

9.17 $$\dot{\mathcal{H}} = 2m\dot{R}\ddot{R} - m\left(2R - L\right)\dot{R}\Omega^2 = 0,$$
$$\dot{E} - \dot{\mathcal{H}} = 2m\left(2R - L\right)\dot{R}\Omega^2 = \Gamma\Omega.$$

9.20 Define $I_1 = m_1\kappa_1^2, \quad I_2 = m_1\kappa_2^2 + \left(m_1 + \frac{1}{3}m_2\right)L^2.$

Case (a) $$\dot{\mathcal{H}} = I_1\dot{\phi}\ddot{\phi} + I_2\dot{\theta}\ddot{\theta} + \frac{1}{2}\left(I_1 - I_2\right)\left(\sin 2\theta\right)\Omega^2\dot{\theta} + \left(m_1 + \frac{1}{2}m_2\right)gL\dot{\theta}\sin\theta = 0,$$
$$\dot{E} = I_1\dot{\phi}\ddot{\phi} + I_2\dot{\theta}\ddot{\theta} - \frac{1}{2}\left(I_1 - I_2\right)\left(\sin 2\theta\right)\Omega^2\dot{\theta} + \left(m_1 + \frac{1}{2}m_2\right)gL\dot{\theta}\sin\theta = \Gamma\Omega.$$

Case (b) $$\dot{\mathcal{H}} = I_2\dot{\theta}\ddot{\theta} + \frac{1}{2}\left(I_1 - I_2\right)\left(\sin 2\theta\right)\Omega^2\dot{\theta} + \left(m_1 + \frac{1}{2}m_2\right)gL\dot{\theta}\sin\theta = 0,$$
$$\dot{E} = I_2\dot{\theta}\ddot{\theta} - \frac{1}{2}\left(I_1 - I_2\right)\left(\sin 2\theta\right)\Omega^2\dot{\theta} + \left(m_1 + \frac{1}{2}m_2\right)gL\dot{\theta}\sin\theta = \Gamma\Omega + \Gamma_\phi\dot{\phi}.$$

9.22 $$\frac{1}{2}mL^2\ddot{\theta}\left[1 + 8\left(\cos\theta\right)^2\right] - 4mL^2\dot{\theta}^2\sin\theta\cos\theta - \frac{2p_2^2}{9mL^2}\frac{\cos\theta}{\left(\sin\theta\right)^3} - mgL\sin\theta = -2FL\cos\theta,$$
$$p_2 = \frac{9}{2}mL^2\dot{\psi}\left(\sin\theta\right)^2.$$

9.25 $$mR^2\ddot{\theta} - \frac{p_2^2\cos\theta}{mR^2\left(\sin\theta\right)^3} + \frac{1}{2}mgR\sin\theta\left[3 - \frac{1}{\left(5 + 4\cos\theta\right)^{1/2}}\right] = 0,$$
$$p_2 = mR^2\dot{\psi}\left(\sin\theta\right)^2 = \frac{3}{4}m\left(gR^3\right)^{1/2}.$$

9.27 $$p_2 = \left[\tilde{I} + I_\phi\left(\cos\phi\right)^2\right]\dot{\psi} - I_\phi\dot{\phi}\cos\beta, \quad p_3 = I_\phi\left(\dot{\phi} - \dot{\psi}\cos\beta\right)$$
$$\mathcal{R} = \frac{1}{2}\left(mL^2 + \frac{1}{2}I_\phi\right)\dot{\beta}^2 - \frac{1}{2I_\phi\tilde{I}}\left[I_\phi\left(p_2 + p_3\cos\beta\right)^2 + \tilde{I}p_3^2\right] + mgL\sin\beta$$
$$\tilde{I} = mL^2\left(1 + \cos\beta\right)^2 + \frac{1}{2}I_\phi\left(\sin\beta\right)^2 + I_P$$

9.30 $$\dot{\gamma}_1 = \dot{\psi}, \quad \dot{\gamma}_2 = \dot{\theta}$$
$$mL^2\sin\theta\left[\frac{1}{2}\ddot{\gamma}_1\sin\theta + 2\dot{\gamma}_1\dot{\gamma}_2\cos\theta\right] = 0$$
$$\left[\frac{1}{2} + 4\left(\cos\theta\right)^2\right]\ddot{\gamma}_2 - mL^2\left(\frac{9}{2}\dot{\gamma}_1^2 + 4\dot{\gamma}_2^2\right)\sin\theta\cos\theta = \frac{(mg - 2F)}{ML}\sin\theta$$

9.32 $$\dot{\gamma}_1 = v, \quad \dot{\gamma}_2 = \dot{\theta}, \quad \dot{X}_G = \dot{\gamma}_1\cos\theta, \quad \dot{Y}_G = \dot{\gamma}_1\sin\theta, \quad m\ddot{\gamma}_1 = F\cos\beta, \quad I\ddot{\gamma}_2 = -FD\sin\beta.$$

9.34 $\left[\frac{1}{3}L^2(\cos\theta)^5 - DL(\cos\theta)^4 + D^2\cos\theta\right]\ddot{\gamma}_1 + \left[2D^2 - \frac{1}{2}DL(\cos\theta)^3(\sin\theta)\dot{\gamma}_1^2\right]$
$$= g\left[D - \frac{L}{2}(\cos\theta)^3\right](\cos\theta)^3, \quad \dot{\gamma}_1 = \dot{\theta}.$$

9.37 $\dot{\gamma}_1 = \dot{\psi}, \quad \dot{\gamma}_2 = \dot{\beta}, \quad \dot{\gamma}_1 - c_1\dot{\gamma}_2 - c_2\beta - c_3 = 0,$
$$m\left[L^2 + \kappa_1^2(\sin\beta)^2 + \kappa_2^2(\cos\beta)^2\right]\ddot{\gamma}_1 + 2m\left(\kappa_1^2 - \kappa_2^2\right)\dot{\gamma}_1\dot{\gamma}_2\sin\beta\cos\beta$$
$$-m\kappa_1^2\dot{\gamma}_1\Omega_1\cos\beta = \lambda_1,$$
$$m\kappa_2^2\ddot{\gamma}_1 + m\kappa_1^2\dot{\gamma}_1\Omega_1\cos\beta - m\left(\kappa_1^2 - \kappa_2^2\right)\gamma_1^2\sin\beta\cos\beta = M - c_1\lambda_1.$$

9.39 $\dot{\gamma}_1 = \dot{r}, \quad \dot{\gamma}_2 = \dot{\theta}, \quad \frac{7}{5}\ddot{\gamma}_1 - r\dot{\gamma}_2^2 = 0, \quad \left(r^2 + \frac{2}{5}R^2\right)\ddot{\gamma}_2 + 2r\dot{\gamma}_1\dot{\gamma}_2 = 0.$

9.42 $\dot{\gamma}_1 = \dot{\theta}, \quad \frac{1}{2}\left[1 + 8(\cos\theta)^2\right]\ddot{\gamma}_1 - \left(4\dot{\gamma}_1^2 + \frac{9}{2}\dot{\psi}^2\right)\sin\theta\cos\theta = \frac{(mg - 2F)}{mL}\sin\theta.$

9.45 $\dot{\gamma}_1 = \dot{\beta}, \quad m\kappa_2^2\ddot{\gamma}_1 + m\kappa_1^2\Omega_1\dot{\psi}\cos\beta + m\left(\kappa_2^2 - \kappa_1^2\right)\dot{\psi}^2\sin\beta\cos\beta = M.$

9.48 $\dot{\gamma}_1 L\cos\beta + \dot{\gamma}_2\sin(\theta + \beta) = 0,$
$$\left(\frac{1}{3}m_1L^2 + m_2L^2 + I_2\right)\ddot{\gamma}_1 + \left(\frac{1}{2}m_1 + m_2\right)L\ddot{\gamma}_2\sin\theta + I_2\ddot{\beta} = F\sin\beta + N_B\cos\beta,$$
$$\left[m_1 + m_2 + m_3 + \frac{I_1}{R^2}\left(\frac{\cos\theta}{\cos\beta}\right)^2\right]\ddot{\gamma}_2 + \left(\frac{1}{2}m_1 + m_3\right)L\left(\ddot{\gamma}_1\sin\theta + \dot{\gamma}_1^2\cos\theta\right)$$
$$-\frac{I_1}{R^2}\dot{\gamma}_1\dot{\gamma}_2\frac{\sin\theta\cos\theta}{(\cos\beta)^2} + \frac{I_1}{R^2}\dot{\gamma}_2\dot{\beta}\frac{(\cos\theta)^2\sin\theta}{(\cos\beta)^3}$$
$$= -F\cos(\beta + \theta) - N_B\sin(\theta + \beta) + k(\ell - \ell_0), \quad \dot{\gamma}_1 = \dot{\theta}, \quad \dot{\gamma}_2 = \dot{\ell}.$$

9.50 $\dot{X}_A = \dot{\gamma}_1\cos(\theta + \beta), \quad \dot{Y}_A = \dot{\gamma}_1\sin(\theta + \beta), \quad v_A\sin\beta - L\dot{\theta} = 0$
$$\left[m_1\left(1 + \frac{\kappa_1^2}{R^2}\right) + m_C\right]\dot{v}_A - m_Ch\ddot{\theta}\sin\beta + m_Ch\dot{\theta}^2\cos\beta = \frac{\Gamma}{R_1} + (N_B + N_C),$$
$$\left(\frac{1}{2}m\kappa_1^2 + m_Ch^2 + I_C\right)\ddot{\theta} + \frac{1}{2}m\kappa_1^2\ddot{\beta} - m_Ch\left[\dot{v}_A\sin\beta + v_A\left(\dot{\theta} + \dot{\beta}\right)\cos\beta\right] = -(N_B + N_C)L,$$
$$\frac{1}{2}m\kappa_1^2(\ddot{\gamma}_2 + \ddot{\gamma}_3) = \Gamma.$$

10.1 $|\bar{\omega}_1| = 3.135$ rad/s, $\theta = 86.96°$, $\Delta\bar{H}_G = -3.130I'\bar{\imath}$, $|\bar{\omega}_2| = 0.1667$ rad/s.

10.3 $\omega_x = 50$ rad/s, $\omega_z = 450$ rad/s, $\beta = 83.66°$, $\theta = 86.82°$.

10.6 $\bar{\omega} = 7.2902\left(10^{-5}\right)$ rad/s about an axis through the center of the earth at 0.1558° from the Polar axis in the meridional plane at 90° from the meridian of impact.

10.8 Precession about the z azis; $\bar{\omega} = 7.594\bar{\imath} + 6.275\bar{k}$ rad/s @ $\max(\theta) = 45.91°$;
$\bar{\omega} = 5.534\bar{\imath} + 8.766\bar{k}$ rad/s @ $\min(\theta) = 13.57°$.

10.11 Looping precession, $\min(\theta) = 53.130°$, $\max(\theta) = 53.300°$, $\dot{\psi} = 0$ at $\theta = 53.199°$.

10.14 $\psi = A_1\cos\left(\frac{\omega^2}{\sigma}t + \nu_1\right) + A_2\cos(\sigma t + \nu_2), \quad \omega^2 = \frac{mgL}{I'}, \quad \sigma = \frac{I}{I' + mL^2},$
$$\theta = \frac{\pi}{2} + A_1\sin\left(\frac{\omega^2}{\sigma}t + \nu_1\right) - A_2\sin(\sigma t + \nu_2).$$

10.16 $\frac{I'\lambda}{c} \gg 1, \quad t \gg \frac{I'\Omega_1}{c\Omega_0}, \quad \frac{I'}{I} = \mathrm{O}(1).$

10.19 $\left[(I + C)(\cos\theta)^2 + (I' + A)(\sin\theta)^2 + A\right]\dot{\psi} + I\dot{\phi}\cos\theta = p_\psi,$
$$I\left(\dot{\psi}\cos\theta + \dot{\phi}\right) = p_\phi, \quad (I' + B)\ddot{\theta} + (I + C - I' - A)\dot{\psi}^2\sin\theta\cos\theta + I\dot{\phi}\dot{\psi}\sin\theta = 0.$$